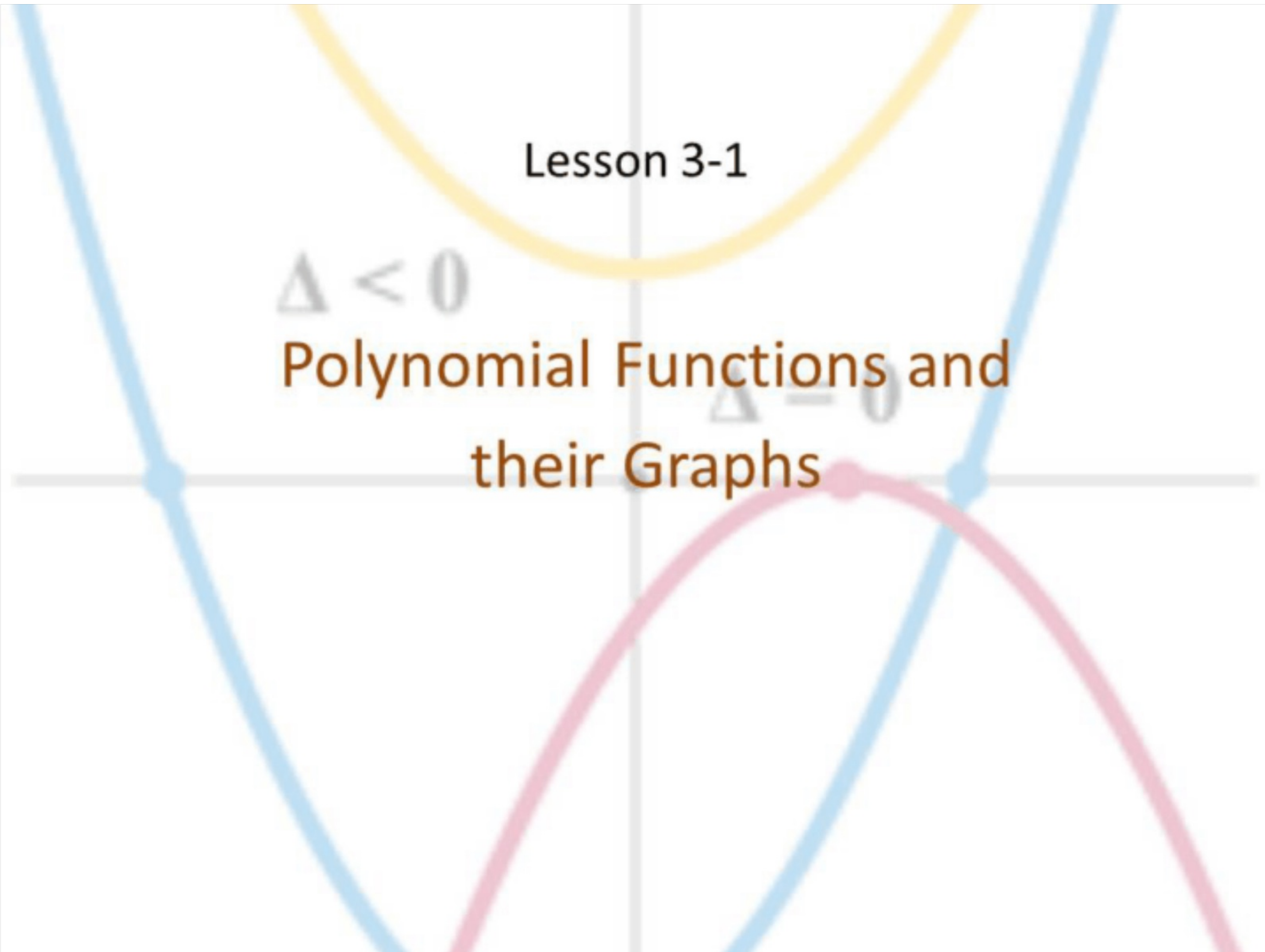
Lesson 3-1

Polynomial Functions and their Graphs



Objective

Students will…

- Be able to define and identify the characteristics of polynomials.
- Be able to find the x (zeros) and the y intercepts of polynomials by factoring, grouping, and using the quadratic formula.

Polynomial Functions

A polynomial function consists of polynomials, which take the form

$$P(x) = \boxed{a_n}x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1x^1 + a_0x^0,$$

where *n* is a nonnegative integer and $a_n \neq 0$.

ex. $2x^2 + x^1 - 1x^0$

The numbers $a_1, a_2, \ldots, a_n$ are called the coefficients.

ex. $-x - 1x^{2+}$

The number a_0 is the constant coefficient or constant term.

The number a_n, the coefficient of the highest power, is the leading coefficient, and the term a_nx^n is the leading term.

Example

Underline each coefficient, circle the constant term (coefficient), and box the leading term of the following polynomial function.

$$P(x) = \boxed{3x^5} + 6x^4 - 2x^3 + 1x^2 + 7x - 6$$

The function *P(x)* above is a polynomial of degree 5.

Polynomials

Here are other examples of different polynomials. Identify the degree of each polynomial.

	Degree
$P(x) = 3x^0$	0
$Q(x) = 4x - 7$	1
$R(x) = x^2 + x$	2
$S(x) = 2x^3 - 6x^2 - 10$	3

Polynomials with just a single term like *P(x)* is called a <u>monomial.</u>

Finding X, Y Intercepts

Finding the x and the y intercepts is an important step in analyzing polynomials. We will also use them for graphing in our next lesson.

To find y-intercept, we set $x = 0$ and find *y*.
To find x-intercept, we set $y = 0$ or $\mathrm{P}(x) = 0$ and find *x*.

Ex. Find the x and the y intercepts of $f(x) = 2x^2 - 1$

y-int: $f(0) = 2(0)^2 - 1 = \boxed{-1}$

x-int: $0 = 2x^2 - 1 \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow \boxed{x = \pm\sqrt{1/2}}$

X-intercepts

As we studied back in Algebra, there's a lot more to x-intercepts. We've learned that the x-intercepts are also known as <u>roots</u> or <u>zeros</u> of the function. All in all, the following are equivalent.

X

1. x-intercepts 
2. Zeros or roots
3. Solutions to the polynomial equations
4. (*) If c is the zero of a polynomial, then $(x - c)$ is one of its factors.

ex. $x^2 - 5x + 6 = 0$
$(x-3)(x-2) = 0$
$x = 3, 2$

With that said, when you are instructed to find real zeros of a function, you are to find the x-intercepts.

Examples

Find the zeros of the following polynomials.

1. $P(x) = (x-2)(x+3)$

$0 = (x-2)(x+3)$

$\boxed{x = 2, -3}$

2. $Q(x) = (x+2)(x-1)(x-3)$

$\boxed{x = -2, 1, 3}$

3. $R(x) = x^3 - 2x^2 - 3x$

$0 = x(x^2 - 2x - 3)$

$0 = x(x+1)(x-3)$

$\boxed{x = 0, -1, 3}$

4. $P(x) = -2x^3 - x^2 + x$

$0 = -x(2x^2 + x - 1)$

$0 = -x(2x-1)(x+1)$

$-x = 0$

$\boxed{x = 0}$

$\boxed{x = 1/2, -1}$

$x^4 = (x^2)^2$?

5. $Q(x) = x^3 + 3x^2 - 4x - 12$

$0 = x^2(x+3) - 4(x+3)$

$0 = (x+3)(x^2-4) = (x+3)(x+2)(x-2)$

$\boxed{x = -3, \pm 2}$

6. $R(x) = (2x^4 + 3x^3 - 16x - 24) = 0$

$2x^4 + 3x^3 - 16x - 24 = 0$

$x^3(2x+3) - 8(2x+3) = 0$

$(x^3-8)(2x+3) = 0$

$\boxed{x = -3/2, \quad 2}$

7. $S(x) = x^4 - 3x^2 - 4$

$= (x^2)^2 - 3(x^2) - 4$

$\sqrt{x^2} = \sqrt{-1}$

$-4 \quad -4 \quad 1 \quad -3$

$0 = (x^2-4)(x^2+1)$

$\boxed{x = \pm 2},$

8. $Q(x) = 7b^2 - 7b + 10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \quad 7 \pm \sqrt{49 - 4(7)(10)}$$

9. $R(x) = 2x^2 - 4x - 11$

$$x = \frac{4 \pm \sqrt{16 - 4(2)(-11)}}{4} = \boxed{\frac{4 \pm \sqrt{104}}{4}} = 1 \pm \frac{\sqrt{104}}{4}$$

Homework 10/10

Zeros of Polynomial WKSHT

$$\left(\frac{\frac{b}{a}}{2}\right)^2 = \left(\frac{b}{2a}\right)^2$$

$$= \frac{b^2}{4a^2}$$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + \frac{c}{a} = \frac{b^2}{4a^2}$$

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 + \cancel{\frac{c}{a}} \cancel{- \frac{c}{a}} = \frac{b^2}{4a^2} - \frac{c \cdot 4a}{a \cdot 4a} = \frac{b^2-4ac}{4a^2}$$

$$\Rightarrow \sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2-4ac}{4a^2}} \Rightarrow x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$\Rightarrow x + \cancel{\frac{b}{2a}} \cancel{- \frac{b}{2a}} = \pm \frac{\sqrt{b^2-4ac}}{2a} - \frac{b}{2a} \Rightarrow \boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$