Warm Up 9/29 $\sqrt{0} = 0$

1. Find the domain of the function

a. $f(x) = x^2 + 4x - 1$

D: $(-\infty, \infty)$

b. $g(x) = \sqrt{9 - x^2}$

$9 - x^2 \geq 0$
$+\cancel{x}^2 \quad +x^2$

$\sqrt{9} \geq \sqrt{x^2}$

$\pm 3 \geq x \Rightarrow$

$3 \geq x$

$-3 \leq x$

$[-3, 3]$

c. $f(x) = \frac{1}{x^2 - x}$

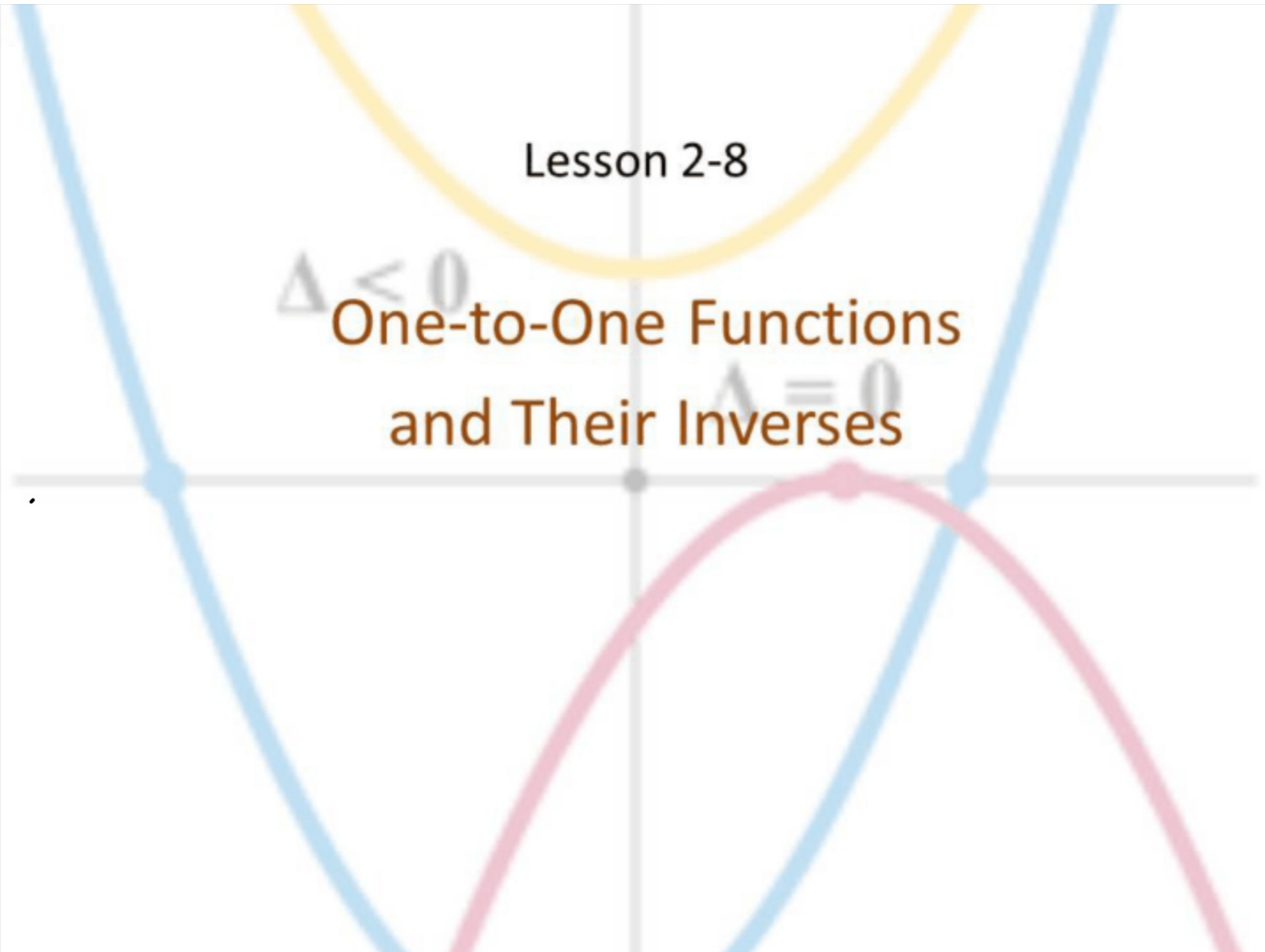
$x^2 - x \neq 0$

$x(x-1) \neq 0$ $\quad x-1 \neq 0$

$x \neq 0, x \neq 1$

$\{x \mid x \neq 0, x \neq 1\}$

$(-\infty, 0) \cup (0, 1) \cup (1,$


Lesson 2-8
Δ < 0
One-to-One Functions
Δ = 0
and Their Inverses

Objective

Students will…

- Be able to define one-to-one functions.
- Be able to prove whether a given function is one-to-one, using horizontal line test and algebraically.
- Be able to find the inverse function of one-to-one functions.

One-to-One Functions

"an"

Function is defined as a relation having one output, per input. This only deals with the **number** of outputs, not necessarily the **type** of outputs. A one-to-one function is a function where no input shares a same output with another input. In other words,

$$f(x_1) = f(x_2) \text{ if and } \textbf{only} \text{ if } x_1 = x_2$$

or

$$f(x_1) \neq f(x_2) \text{ if and } \textbf{only} \text{ if } x_1 \neq x_2$$

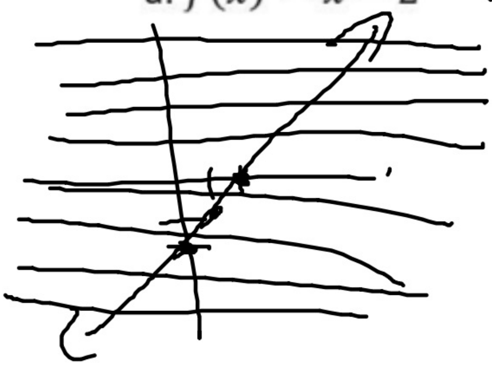
Again, the definition of a function only deals with the number of outputs. Two different inputs could share the same output, as long as they both have **one** single output.

Horizontal Line Test

One of the ways to visually identify one-to-one functions is using the horizontal line test, which simply states that a function is one-to-one if and only if no horizontal line intersects its graph more than once.

Ex. Use the horizontal line test to determine whether the following functions are one-to-one.

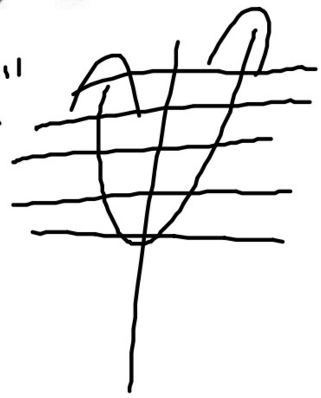
a. $f(x) = x - 2$ "1 to 1"



b. $f(x) = x^2$

x	x^2
-2	4
-1	1
0	0
1	1
2	4

Not "1 to 1"



Proving One-to-one-"ness"

Sometimes horizontal line test may not be suitable, because drawing the graph of a function may be difficult. In this case, we need to determine whether a function is one-to-one algebraically. $f(x_1)=f(x_2)$ iff $x_1=x_2$.

Ex. Show that the following function is one-to-one

1. $f(x) = 3x + 4$

$f(x_1)=f(x_2) \Rightarrow 3x_1+\cancel{4} = 3x_2+\cancel{4}$

$-4 \qquad -4$

$\Rightarrow \frac{\cancel{3}x_1}{\cancel{3}} = \frac{\cancel{3}x_2}{\cancel{3}} \Rightarrow x_1 = x_2$ ✓ "1 to 1"

2. $g(x) = x^2$

$g(x_1)=g(x_2) \Rightarrow \sqrt{x_1^2} = \sqrt{x_2^2}$

$\Rightarrow \pm x_1 = \pm x_2$ Not "1 to 1"

$\Rightarrow x_1 \neq x_2$

3. $f(x) = x^3$

$f(x_1)=f(x_2) \Rightarrow \sqrt[3]{x_1^3} = \sqrt[3]{x_2^3}$

$\Rightarrow x_1 = x_2$ ✓ "1 to 1"

4. $h(x) = 4x^4$

$h(1) = 4 \quad 1 \neq -1$

$h(-1) = 4$

Not "1-1"

Examples

Algebraically, show whether the following functions are one-to-one

a. $f(x) = x + 2$ b. $g(x) = 6x^2 - 4$ $f(x) \to$ c. $9x^3 - 7$

$f(x_1) = f(x_2)$

$\Rightarrow x_1 + \cancel{2} = x_2 + \cancel{2}$

$-2 \qquad -2$

$x_1 = x_2$ ✓

"1 to 1"

$g(1) = 6 - 4 = 2$

$g(-1) = 6 - 4 = 2$

$1 \neq -1$

Not "1 to 1"

$f(x_1) = f(x_2)$

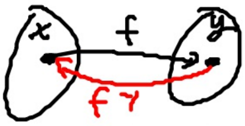
$\Rightarrow \frac{\cancel{9} x_1^3 \cancel{-7}}{\cancel{9}} = \frac{\cancel{9} x_2^3 \cancel{-7}}{\cancel{9}}$

$+7 \qquad +7$

$\Rightarrow \sqrt[3]{x_1^3} = \sqrt[3]{x_2^3}$

$\Rightarrow x_1 = x_2$ ✓

"1 to 1"



Inverse Functions

The whole point of finding out whether a function is one-to-one or not has to do with inverse functions. For any one-to-one function, an inverse function must exist.

Inverse functions is the "opposite" function. By definition, for a function f, let $\underset{\text{in}}{f(x)} = \underset{\text{out}}{y}$. Then, the inverse function f^{-1},

ex. $f(x) = x^2$

$f(1) = 1$ $\quad$ $f(1) = 1$

$f(-1) = -1$ $\quad$ $f(-1) = 1$

$f^{-1}(\overset{\text{out}}{y}) = \overset{\text{in.}}{x}$, for any y.

You can also think inverse function as the function that "undo's" the its original function.

Example

Assume f is one-to-one…

If $f(5) = 18$, find $f^{-1}(18) = 5$ If $f^{-1}(3) = 6$, find $f(6) = 3$

$f(6) = 16$

If $f(x) = 3x - 2$, find $f^{-1}(16) = 6$

$16 = 3x - 2$

$18 = 3x$

$6 = x$

How to find the inverse function

1. Write "y =" instead of "f(x) ="

2. Replace ~~the~~ or switch the "y" and the "x"

3. Solve the equation for "y"

4. The resulting equation is the inverse function, $f^{-1}(x)$

Example

1. $f(x) = x^3$

1. $y = x^3$
2. $x = y^3$
3. $y = \sqrt[3]{x}$
4. $\boxed{f^{-1}(x) = \sqrt[3]{x}}$

2. $f(x) = x + 1$

1. $y = x + 1$
2. $x = y + 1$
3. $y = x - 1$
4. $\boxed{f^{-1}(x) = x - 1}$

3. $f(x) = 2x - 3$

1. $y = 2x - 3$
2. $x = 2y - 3$
3. $y = \frac{x+3}{2}$
4. $\boxed{f^{-1}(x) = \frac{x+3}{2} = \frac{1}{2}x + \frac{3}{2}}$

Example

Remember! Only one-to-one functions can have an inverse function. So, it's important to make sure that the function is one-to-one before you try to find its inverse.

I.E. $f(x) = x^2$ does not have an inverse function!

$y = x^2$

$x = y^2$

$y = \pm\sqrt{x}$

~~$f^{-1}(x) = \pm\sqrt{x}$~~

x	$\pm\sqrt{x}$
1	1, -1
4	2, -2

Homework 9/29

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#1-13 (e.o.o), 17, 19, 33, 37, 45