

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-2}{2}\right)^2$$
$$= (-1)^2 = 1$$

Warm Up 9/23

$$f(x) = 3(x-1)^2 - 1.$$

1. Complete the square:  $f(x) = 3x^2 - \frac{6x}{3} - \frac{1}{3}$

$$\Rightarrow \frac{f(x)}{3} = x^2 - 2x - \frac{1}{3} \Rightarrow \frac{f(x)}{3} + 1 = x^2 - 2x + 1 - \frac{1}{3} = (x-1)^2 - \frac{4}{3}.$$

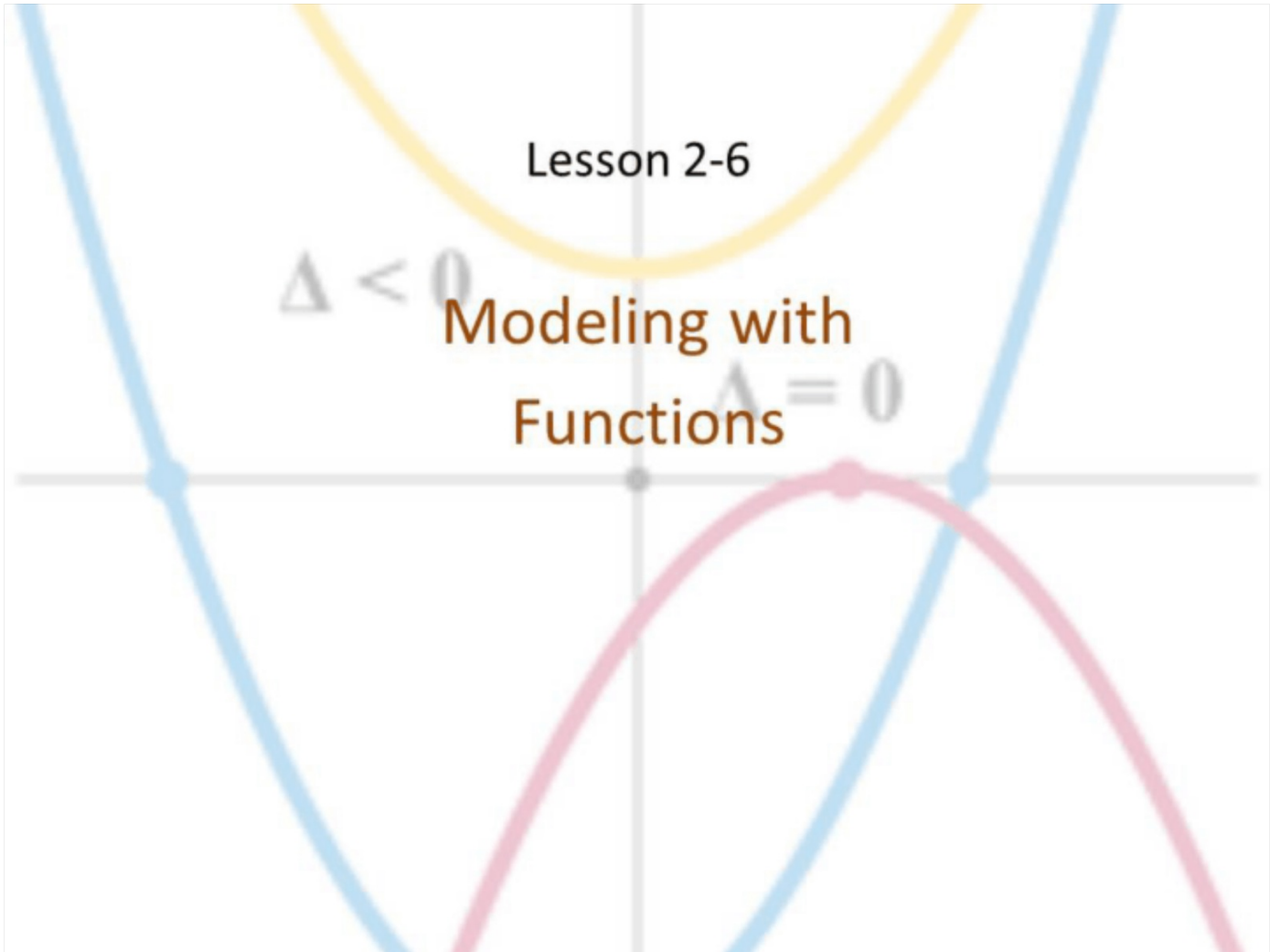
$$\Rightarrow f(x) = 3(x-1)^2 - 4$$

Lesson 2-6

$\Delta < 0$

Modeling with  
Functions

$\Delta = 0$



## Objective

Students will...

- Model real-life word problems using quadratic functions.
- Be able to solve real-life word problems using functions.

## Modeling with Functions

We saw in our previous lesson that quadratic functions can be used to solve real-life related problems, by observing and studying its behavior.

Before, we were given with a function that modeled different situations, although this process is by far the most difficult of all. This section, we learn how to model some of the real-life situations using algebraic and geometric properties.

## Guidelines for Modeling with Functions

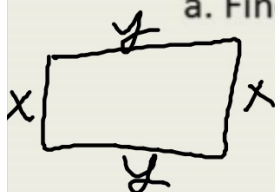
You may use the following guidelines to aid you if you wish...

1. Express the model (formula) in words- Ex. Area = length x width
2. Choose the variable- Identify all the variables used to express the function. Key is writing it all using one variable instead of multiple.
3. Set up the model- Once you have it written all under one variable, write the function in mathematical language.
4. Use the model- Hard work is virtually done! You may use the function model to solve other applicable problems.

= Perimeter.

A gardener has 140ft of fencing to fence in a rectangular vegetable garden.

a. Find a function that models the area of the garden she can fence.



$$A(x) = 70x - x^2$$

$$A = \text{length} \times \text{width.}$$

$$A = xy$$

$$A(x) = x(70 - x).$$

$$A(x) = 70x - x^2.$$

$$x + x + y + y = 140$$

$$2x + 2y = 140$$

⊖

$$x + y = 70$$

$$y = 70 - x$$

b. For what range of widths is the area greater than or equal to  $825\text{ft}^2$ ?

$$15 \leq x \leq 55$$

$$x = -b \pm \sqrt{b^2 - 4ac}$$

c. Can she fence a garden with area  $1250\text{ft}^2$ ?

$$A(x) = 70x - x^2$$

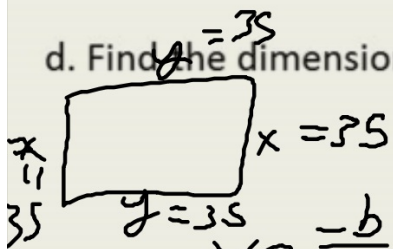
$$70x - x^2 = 1250$$

$$-x^2 + 70x - 1250 = 0$$

No

$$\begin{aligned} & \sqrt{b^2 - 4ac} \\ &= \sqrt{70^2 - 4(-1)(1250)} \\ &= \sqrt{4900 - 5000} \\ &= \sqrt{-100} \end{aligned}$$

d. Find the dimensions of the largest area she can fence.



$$A(x) = 70x - x^2$$

$$x = \frac{-b}{2a} = \frac{-70}{2(-1)} = 35$$

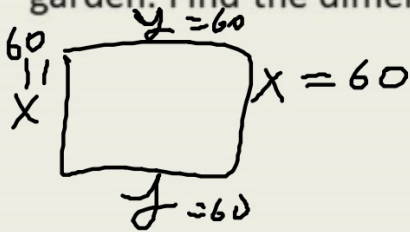
$$y = 35$$





= Perim.

A gardener has 240 feet of fencing to fence in a rectangular vegetable garden. Find the dimensions of the largest area she can fence.



$$2x + 2y = 240$$

$$x + y = 120$$

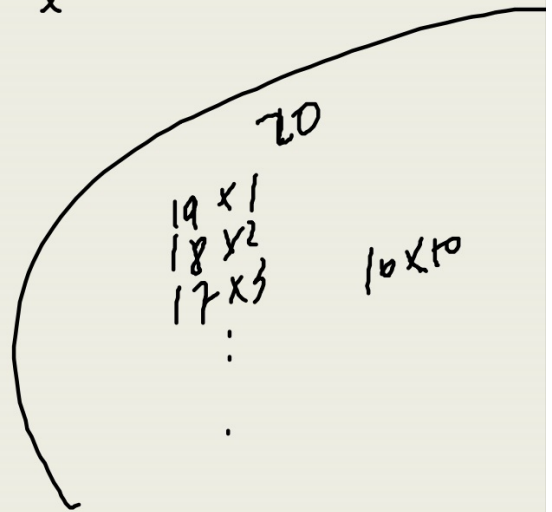
$$y = 120 - x$$

$$A = xy$$

$$A(x) = x(120 - x)$$

$$A(x) = 120x - x^2$$

$$x = \frac{-b}{2a} = \frac{-120}{-2} = 60$$



A hockey team plays in an arena with a seating capacity of 15,000. With the ticket price set a \$14, average attendance at recent games has been 9500. A market survey indicates that for each dollar the ticket price is lowered, the average attendance increases by 1000.

a. Find a function that models the revenue in terms of ticket price.

$x = \text{tix price}$

Revenue = \$Tix  $\times$  Att. ↙ input

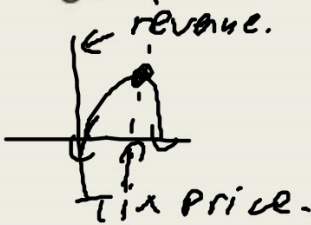
$$R(x) = ((14 - x)1000 + 9500)x$$

$$= (14000 - 1000x + 9500)x = (23500 - 1000x)x$$

$$R(x) = 23500x - 1000x^2$$

$$R(x) = 23500x - 1000x^2$$

b. What ticket price is so high that no one attends, and hence no revenue is generated?



$$0 = 23500x - 1000x^2$$

$$0 = x(23500 - 1000x)$$

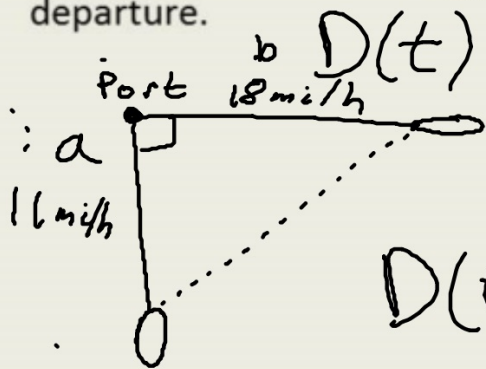
$$x = 0 \quad \text{or} \quad 23500 - 1000x = 0$$
$$23500 = 1000x$$

$$\boxed{\$23.50 = x}$$

c. Find the price that maximizes revenue from ticket sales.

$$x = \frac{-b}{2a} = \frac{-23500}{2(-1000)} = \frac{23.5}{2} = \boxed{\$11.75}$$

Two ships leave port at the same time. One sails south at 11mi/h and the other sails east at 18mi/h. Find a function that models the distance  $D$  between the ships in terms of the time  $t$  (in hours) elapsed since their departure.



$$b = 18t$$

$$a = 11t$$

$t = \text{time in hrs.}$

$$a^2 + b^2 = c^2$$

$$c = \sqrt{a^2 + b^2}$$

$$\text{distance} = \sqrt{a^2 + b^2}$$

$$D(t) = \sqrt{(11t)^2 + (18t)^2}$$

$$= \sqrt{121t^2 + 324t^2}$$

$$D(t) = \sqrt{445t^2} = t\sqrt{445}$$


$$(100-x)(100-x)$$

Find two positive numbers whose sum is 100 and the sum of whose squares is a minimum.

function  $x + y = 100$   
 $y = 100 - x$

$$f = x^2 + y^2$$

$$f(x) = x^2 + (100-x)^2$$

sum  $\rightarrow$    
 $= x^2 + 10000 - 200x + x^2$

$$f(x) = 2x^2 - 200x + 10000$$

$$x = \frac{-b}{2a} = \frac{200}{2(2)} = 50$$

ex. 100  
 $20^2 + 80^2 = 6800$

$$30^2 + 70^2 = 5800$$

$$f(30) = (30)^2 - 200(30) + 10000$$

$$1800 - 6000 + 10000$$
$$-4200 + 10000$$

$$= 5800$$

$$50^2 + 50^2 = \text{min.}$$
$$49^2 + 51^2 =$$
$$f(x) = 2^x$$
$$2^{30} =$$

## Homework 9/23

TB pg. 210-211 #1, 3, 7, 9, 11, 13,  
21, 23, 24, 27