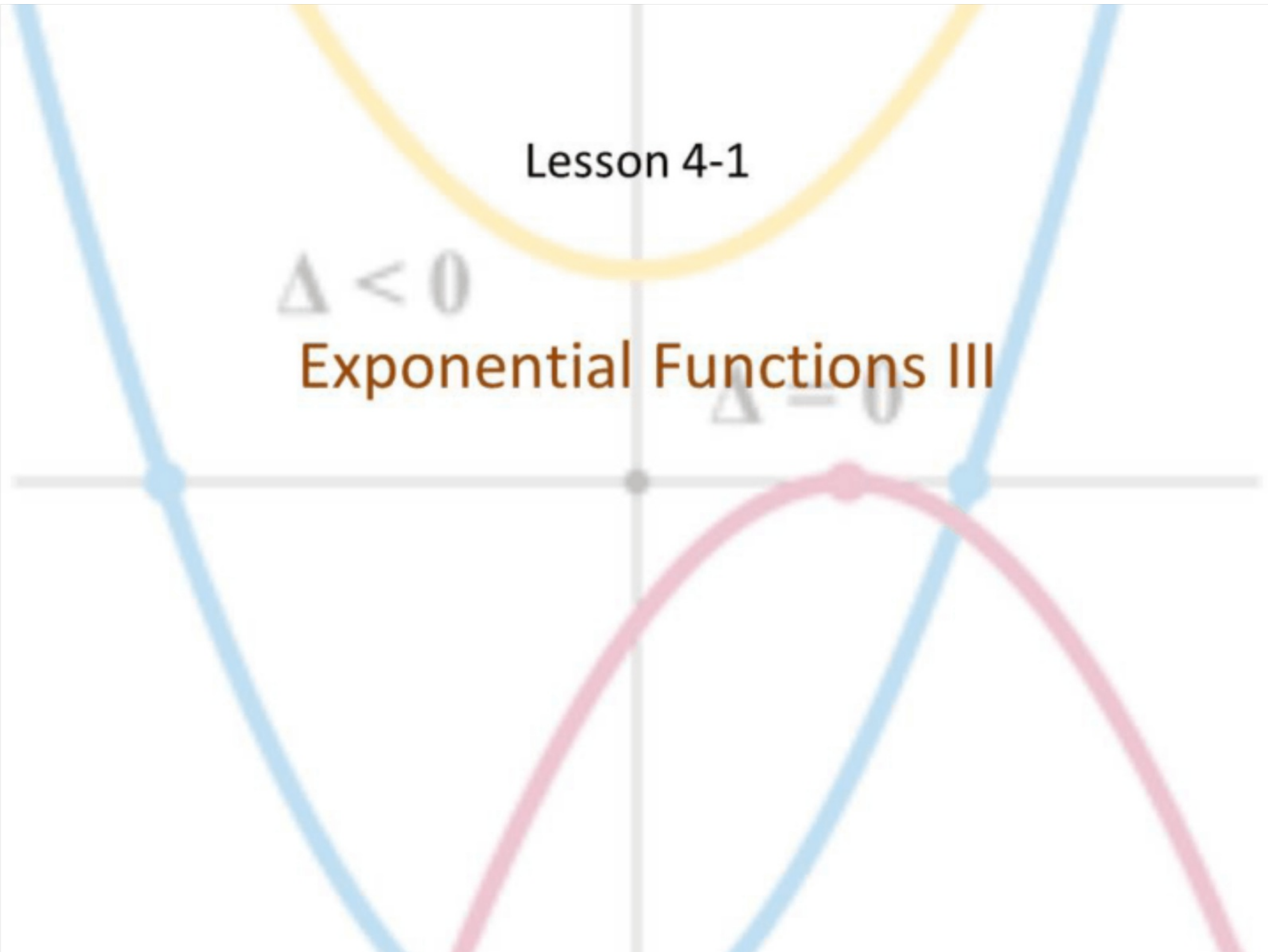
Lesson 4-1

Exponential Functions III



Objective

Students will…

- Be able to solve (compounded)interest problems using the natural exponential function, $f(x) = e^x$

Exponential Functions

Leonard Euler.

In our previous chapter, we studied polynomial and rational functions. Yet another important and practical function group is the exponential function.

The **exponential function** with **base** *a* is defined for all real numbers by

$$f(x) = a^x, \text{ where } a > 0 \text{ and } a \neq 1.$$

Also, recall the ***natural* exponential function**, which is the exponential function

$$f(x) = e^x, \text{ where the base } e \approx 2.71828 \ldots$$

Compound Interest $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

The question was why use such a strange and random base? Actually, it turns out that this little e has much use out in the real world. Again, in Calculus you will see that e isn't all that "random," and have a better idea why e has so much use out in the real world. Here's an example: Calculating Compound Interest! Ka-ching!

Compound Interest is calculated by the formula: $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$

where $A(t) =$ the amount after t years

$P =$ the Principal amount (initial amount put in)

$r =$ the interest rate per year $/100$

* $n =$ the number of times interest is compounded per year

$t =$ the number of years

$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$

Example

A sum of \$1000 is invested at interest rate of 12% per year. Find the amount in the account after 3 years if interest is compounded annually. Quarterly?

Here we need to use our compound interest formula.

Annually.

$P = 1000$
$r = 0.12$
$t = 3$
$n = 1$

$A(3) = 1000\left(1 + \frac{0.12}{1}\right)^{(1 \cdot 3)} \approx \boxed{\$1404.92}$

Quarterly

$P = 1000$
$r = 0.12$
$t = 3$
$n = 4$

$A(3) = 1000\left(1 + \frac{0.12}{4}\right)^{(4 \cdot 3)}$

$\approx \boxed{\$1425.76}$

Continuously Compounded Interest

For some accounts, the interest is compounded continuously, rather than periodically (like our previous problem). For this kind of compounding, the formula is actually much simpler.

<u>Continuously Compounded Interest</u> is calculated by,

$$A(t) = Pe^{rt} \quad \text{where,}$$

$A(t) =$ the amount after t years

$P =$ the Principal amount (initial amount put in)

$r =$ the interest rate per year

$t =$ the number of years

Example

Find the amount after 3 years if $1000 is invested at an interest rate of 12% per year, compounded continuously.

$P = 1000$
$r = 0.12$
$t = 3$

$A(t) = Pe^{rt}$

$A(3) = 1000e^{(0.12 \cdot 3)}$

$\approx \boxed{\$1433.32}$

$\cancel{1000e^{(0.12)(3)}}$

$\cancel{1000 \times e^{(0.12)(3)}}$

A sum of \$3000 is invested. What is the balance in the account and the amount of interest after 4 years if you earn: $A(t) = P\left(1+\frac{r}{n}\right)^{nt}$.

a. 1.9% interest compounded annually?

$r = 0.019$

$A(4) = \$3234.58$

b. 1.6% compounded monthly?

$A(4) = \$3198.14$

A sum of $3000 is invested. What is the balance in the account and the amount of interest after 4 years if you earn:

c. 1.4% compounded daily?

$A(3) = \$3172.78$

d. 0.765% compounded continuously?

$A(3) = \$3093.21$

$r = .00765$

In Closing

Explain to your neighbor the difference between continuous and compound interest.

Homework 11/7

Compound Interest WKSHT