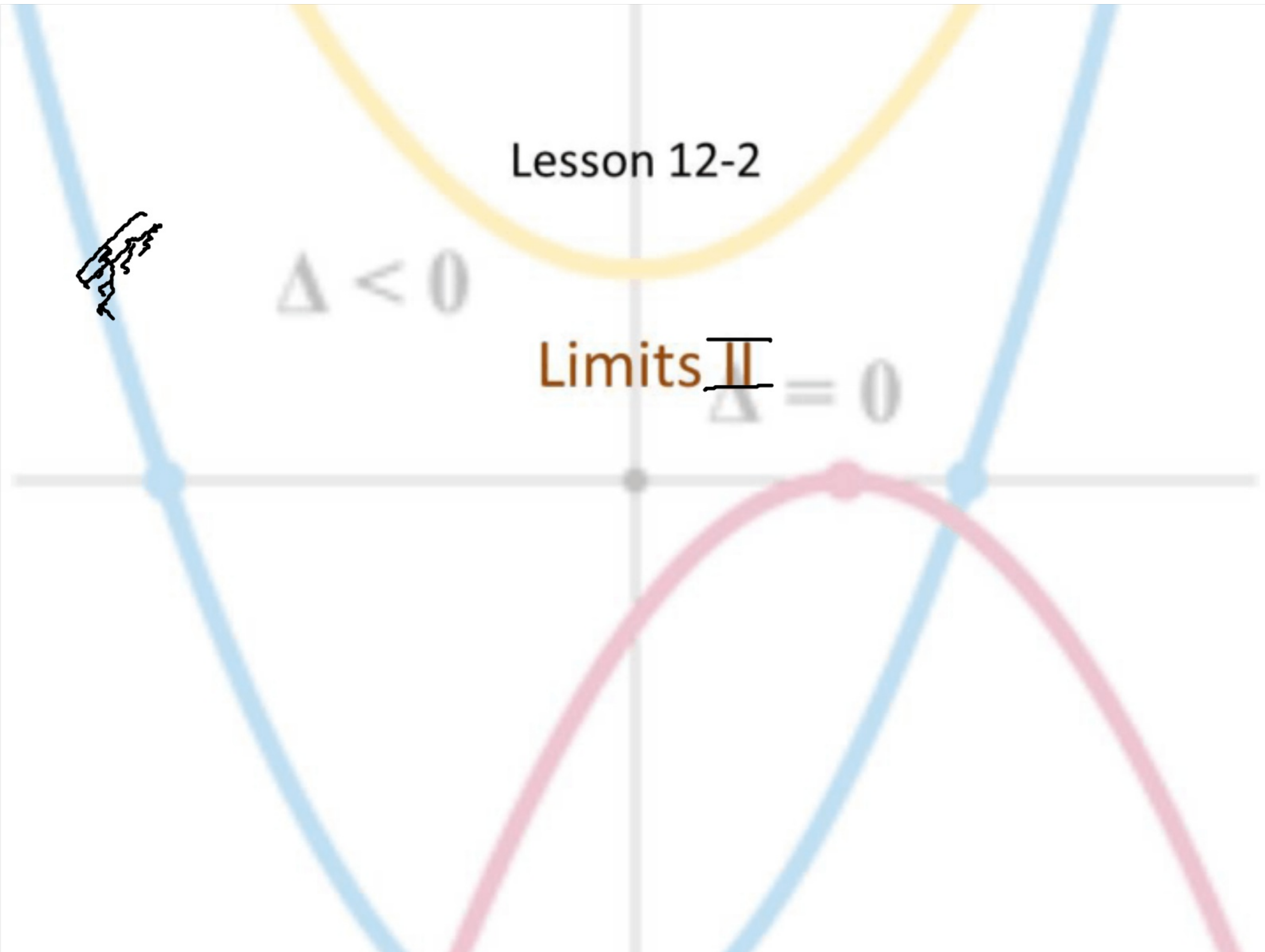
Lesson 12-2

Limits II



Objective

Students will…

- Be able to know and use the Limit Laws.
- Be able to find limits by direct substitution.

Definition of Limit

Let's start by defining limit mathematically.

<u>Definition of the Limit of a Function</u>:

We write, $\lim_{x \to a} f(x) = L$, and say, "the limit of $f(x)$, as x approaches a, equals L," if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to a, but not equal to a.

In other words, this says that the values of $f(x)$ get closer and closer to the number L as x gets closer and closer to the number a (from either side of a) but $x \neq a$.

Alternative notation for $\lim_{x \to a} f(x) = L$ is $f(x) \to L$ as $x \to a$

Limit Laws

In this section we seek to find limits algebraically. First off, let's go ahead and look at some properties of limits, called the **Limit Laws**.

Limit Laws

Suppose that c is a constant and that the following limits exist:

$$\lim_{x \to a} f(x) \quad \text{and} \quad \lim_{x \to a} g(x)$$

f: x + 2
g: x² − 3x +

Then

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$ — Limit of a Sum
2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$ — Limit of a Difference
3. $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$ — Limit of a Constant Multiple
4. $\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$ — Limit of a Product
5. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$ — Limit of a Quotient

Limit Laws Cont.

Limit Laws

6. $\lim_{x \to a} [f(x)]^n = [\lim_{x \to a} f(x)]^n$ where n is a positive integer — Limit of a Power

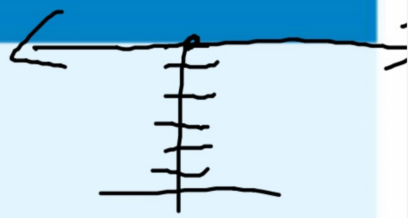
7. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$ where n is a positive integer — Limit of a Root

[If n is even, we assume that $\lim_{x \to a} f(x) > 0$.]

Some Special Units

1. $\lim_{x \to a} c = c$

ex. $\lim_{x \to 2} 6 = 6$



2. $\lim_{x \to a} x = a$

$x \cdot x \cdot x$

3. $\lim_{x \to a} x^n = a^n$ where n is a positive integer

4. $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer and $a > 0$

Example

Evaluate the following limits without making a table or using a graph.

a. $\lim_{x\to 5}(2x^2 - 3x + 4)$ $= \lim_{x\to 5} 2x^2 - \lim_{x\to 5} 3x + \lim_{x\to 5} 4.$

$= 2\lim_{x\to 5} x^2 - 3\lim_{x\to 5} x + 4$

$= 2(5)^2 - 3(5) + 4 = \boxed{39}$

b. $\lim_{x\to -2}\frac{x^3+2x^2-1}{5-3x}$ $= \frac{\lim_{x\to -2} x^3 + \lim_{x\to -2} 2x^2 - \lim_{x\to -2} 1}{\lim_{x\to -2} 5 - \lim_{x\to -2} 3x} = \frac{\lim_{x\to -2} x^3 + 2(\lim_{x\to -2} x^2) - 1}{5 - 3\lim_{x\to -2} x}$

$= \frac{(-2)^3 + 2(-2)^2 - 1}{5 - 3(-2)} = \boxed{-\frac{1}{11}}$

Direct Substitution

Consider the previous problem that we just did

$$\lim_{x \to 5} (2x^2 - 3x + 4)$$

What's interesting here is that $f(5) = 39$, which was indeed the limit from above. It turns out this can be generalized.

Limits by Direct Substitution: If f is a polynomial or a rational function and a is in the domains of f, then

$$\lim_{x \to a} f(x) = f(a)$$

Functions with this property are called **continuous at a**. You will learn more about this in Calculus.

Example

a. $\lim_{x \to -2} \frac{x^3+2x^2-1}{5-3x} = \frac{(-2)^3+2(-2)^2-1}{5-3(-2)} = \frac{-8+8-1}{5+6} = \boxed{\frac{-1}{11}}$

b. $\lim_{x \to 0} \frac{x-2}{x^2+x-6} = \boxed{1/3}$

Example

c. $\lim_{x \to 3} (2x^3 - 10x - 8)$

$= \boxed{16}$

d. $\lim_{x \to -1} \frac{x^2+5x}{x^4+2} = \boxed{\frac{-4}{3}}$

e.) $\lim_{x \to 1} \frac{x-1}{x^2-1} = ?$

Homework 5/27

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