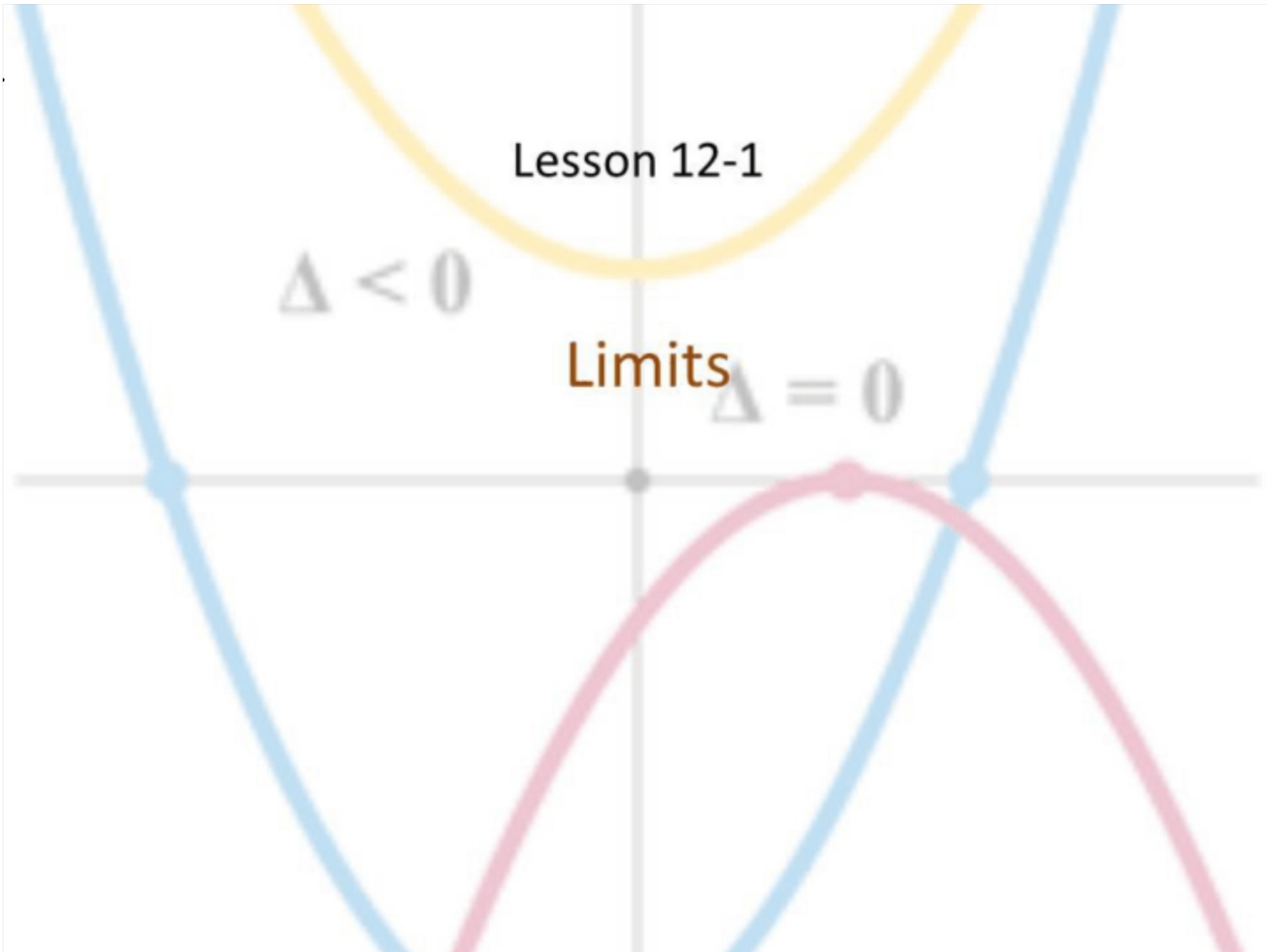
Lesson 12-1

Limits



Objective

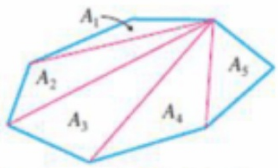
Students will…

- Be able to define limits.
- Be able to estimate limits from numerical tables.
- Be able to estimate limits from a graph.

Limits

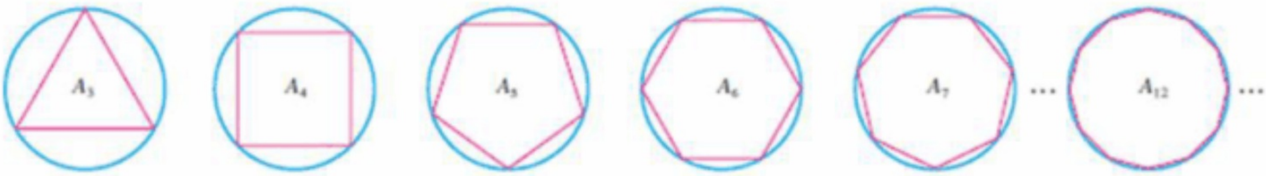
The concept of **limit** is the central idea underlying Calculus. Calculus involves focuses on studying and solving problems pertaining to situations involving **change** or **motion**. To gain a better understanding of limits consider the following example.

Finding the area of a polygon:



$A = A_1 + A_2 + A_3 + A_4 + A_5$

Finding the area of a circle:



$$\text{area} = \lim_{n\to\infty} A_n$$

Definition of Limit

Let's start by defining limit mathematically.

Definition of the Limit of a Function:

We write, $\lim_{x \to a} f(x) = L$, and say, "the limit of $f(x)$, as x approaches a, equals L," if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to a, but not equal to a.

In other words, this says that the values of $f(x)$ get closer and closer to the number L as x gets closer and closer to the number a (from either side of a) but $x \neq a$.

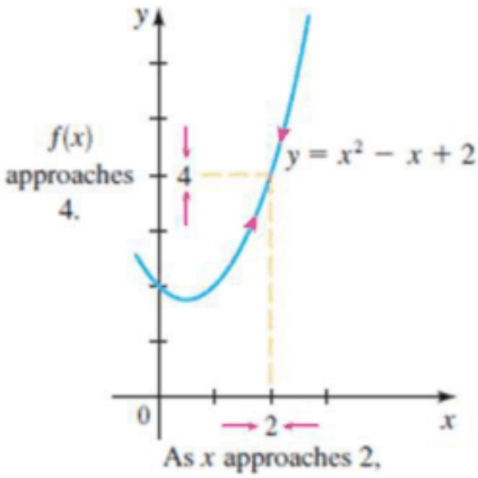
Alternative notation for $\lim_{x \to a} f(x) = L$ is $f(x) \to L$ as $x \to a$

Example

Consider the following function: $f(x) = x^2 - x + 2$. Here are the tables and graph concerning this function, with x surrounding 2, but not equal to 2.

x	$f(x)$
1.0	2.000000
1.5	2.750000
1.8	3.440000
1.9	3.710000
1.95	3.852500
1.99	3.970100
1.995	3.985025
1.999	3.997001

x	$f(x)$
3.0	8.000000
2.5	5.750000
2.2	4.640000
2.1	4.310000
2.05	4.152500
2.01	4.030100
2.005	4.015025
2.001	4.003001



$$\lim_{x \to 2} x^2 - x + 2 = 4$$

Estimating Limits

Guess the value of $\lim_{x \to 1} \frac{x-1}{x^2-1}$, using the table.

$x < 1$	$f(x)$
0.5	0.666667
0.9	0.526316
0.99	0.502513
0.999	0.500250
0.9999	0.500025

$x > 1$	$f(x)$
1.5	0.4000000
1.1	0.476190
1.01	0.497512
1.001	0.499750
1.0001	0.499975

$\lim_{x \to 1} \frac{x-1}{x^2-1} = 0.5$

Guess the value of $\lim_{x \to 0} \frac{\tan 3x}{x}$ = DNE

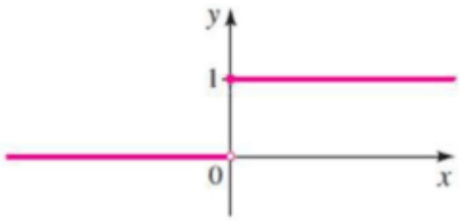
x	$\frac{\tan 3x}{x}$
0.1	3.093362496
0.01	3.000900324
0.001	3.000009000
0.0001	3.000000090
−0.0001	−3.000000090
−0.001	−3.000009000
−0.01	−3.000900324
−0.1	−3.093362496

Limits that Fail to Exist

There are certain occasions where the limit does not exist.

1. A function with a "Jump."

Ex. $$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$



<u>Limit as $t \to 0$ does not exist.</u>

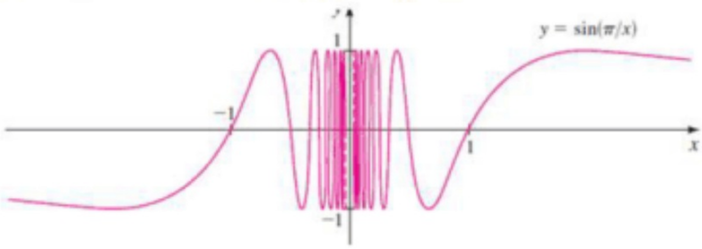
2. A function that oscillates.

Ex. $$\lim_{x \to 0} \sin \frac{\pi}{x}.$$

$f(1) = \sin \pi = 0$ $\quad f(\tfrac{1}{2}) = \sin 2\pi = 0$

$f(\tfrac{1}{3}) = \sin 3\pi = 0$ $\quad f(\tfrac{1}{4}) = \sin 4\pi = 0$

$f(0.1) = \sin 10\pi = 0$ $\quad f(0.01) = \sin 100\pi = 0$



Is it zero? However…

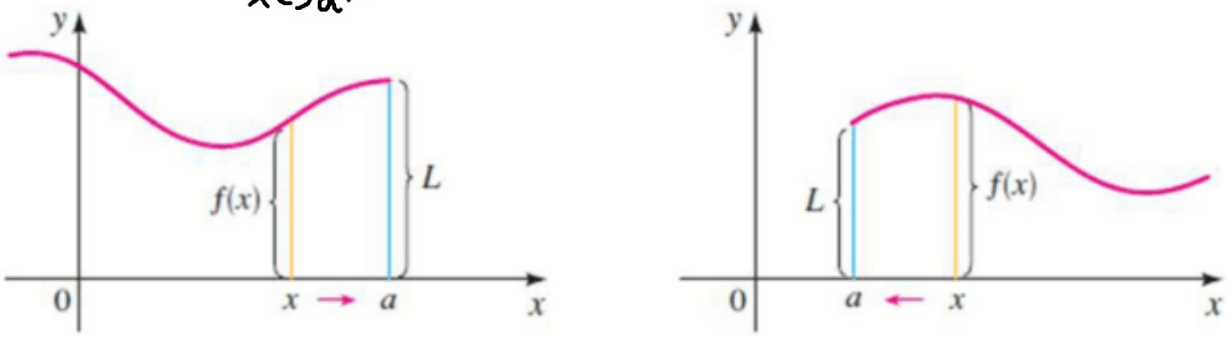
Limit does not exist!

One-Sided Limit

One of the ways we can consider whether a function has a limit or not, is to consider them **one side at a time**.

We write $\lim_{x \to a^-} f(x) = L$, for x approaching a from the left side.

We write $\lim_{x \to a^+} f(x) = L$, for x approaching a from the right side.



(a) $\lim_{x \to a^-} f(x) = L$ (b) $\lim_{x \to a^+} f(x) = L$

Existence of Limit

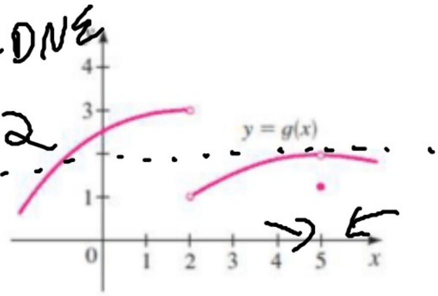
So in comparing the two sides of the limit, we see that the following is true.

$$\lim_{x \to a} f(x) = L \quad \text{if and only if} \quad \lim_{x \to a^-} f(x) = L \quad \text{and} \quad \lim_{x \to a^+} f(x) = L$$

Ex. The graph of a function g is shown in Figure 10. Use it to state the values (if they exist) of the following:

(a) $\lim_{x \to 2^-} g(x)$, $\lim_{x \to 2^+} g(x)$, $\lim_{x \to 2} g(x)$ = 3, 1, DNE

(b) $\lim_{x \to 5^-} g(x)$, $\lim_{x \to 5^+} g(x)$, $\lim_{x \to 5} g(x)$ = 2, 2, 2

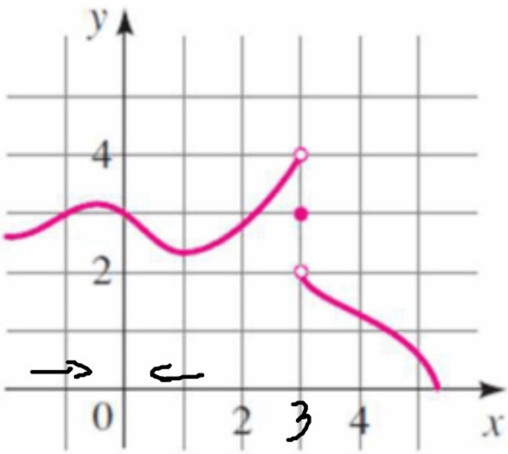


Homework Problem

14. For the function f whose graph is given, state the value of the given quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 0} f(x)$ = 3 **(b)** $\lim_{x \to 3^-} f(x)$ = 4 **(c)** $\lim_{x \to 3^+} f(x)$ = 2

(d) $\lim_{x \to 3} f(x)$ = DNE **(e)** $f(3)$ = 3



$\lim_{x \to 1} g(x)$

f(1) = und.

Homework 5/25

TB pgs. 889-890 #13, 15, [redacted]