Warm Up 1/29

$\frac{\sin C}{c} = \frac{\sin A}{a} \Leftrightarrow \frac{c}{\sin C} = \frac{a}{\sin A}$

Consider
$180 - 57.2 = 122.8$

Use the Law of Sines to solve the triangle.

$a = 26, c = 15, \angle C = 29°$

$\frac{\sin C}{c} = \frac{\sin A}{a} \Rightarrow (26)\frac{\sin 29}{15} = \frac{\sin A}{26}(26)$

$0.84 \approx \sin A$

$\sin^{-1}(0.84) \approx 57.2° = A$ ① $\Rightarrow B = 93.8°$

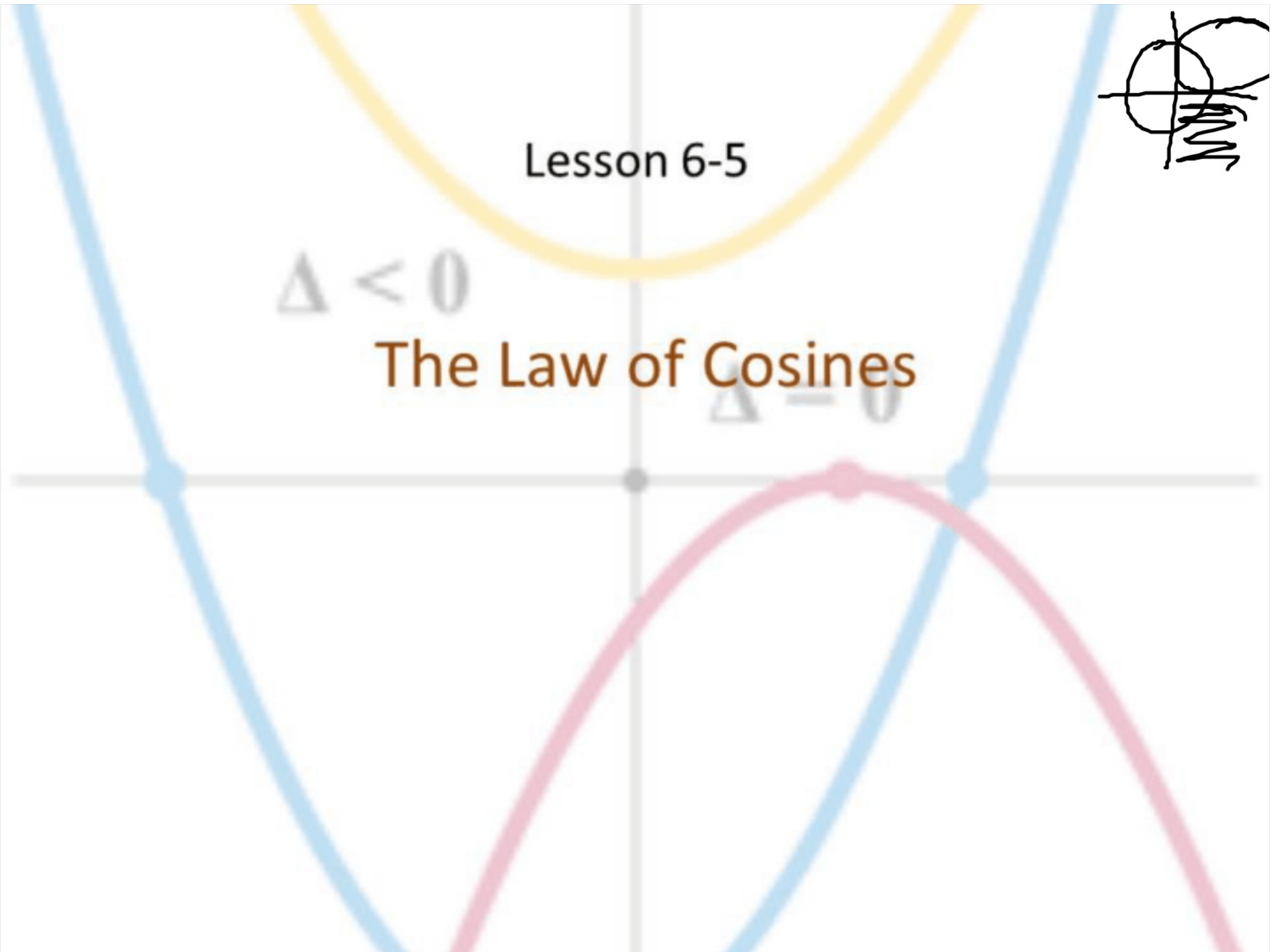
$122.8° = A$ ② $\Rightarrow B = 28.2°$

① $\frac{c}{\sin C} = \frac{b}{\sin B} \Rightarrow \frac{15}{\sin 29} = \frac{b}{\sin 93.8}$ (multiply both sides by $\sin 93.8$)

$30.9 = b$

② $\frac{15}{\sin 29} = \frac{b}{\sin 28.2}$

$14.6 \approx b$


Lesson 6-5
Δ < 0
The Law of Cosines

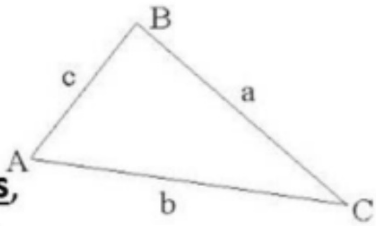
Objective

Students will…

- Be able to know what Law of Cosines is.
- Be able to apply the Law of Cosines to solve for missing sides or angles.

Triangles

We’ve been studying the trigonometric ratios involving right triangles. Trigonometry can also be used for **non**-right triangles. First thing we need to do is to be consistent with our notations.



Consider the triangle ΔABC shown on the right.
The uppercase letters A, B, C represent the **vertices**, or the **angles** of the triangle, while the lower case letters a, b, c represent the sides. For ease, the angles will always be labeled by uppercase letters, while the side **opposite** to each angle, will always be labeled with the lowercase letter of the opposite angle.

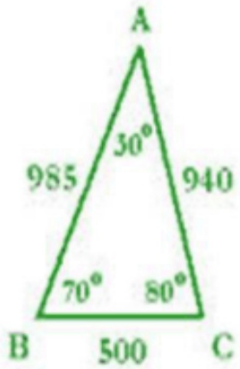
So, from our picture, we see that a is the side opposite to A, while b is the side opposite to B and c is the side opposite to C.

Law of Cosines

There exists another important law regarding triangles (not just right triangles).

<u>Law of Cosines</u>- In any triangle, say, ΔABC, we have:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$



For the ΔABC to the left, we have...

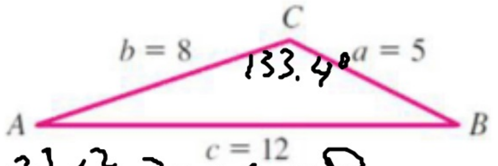
$$500^2 = 940^2 + 985^2 - 2(940)(985)\cos 30$$

Example

So we can apply the Law of Cosines to solve for missing sides or angles.

(Important: Make sure your calculator is in the right mode!)

Find the angles of the triangle.



$b^2 = a^2 + c^2 - 2ac\cos B$

$8^2 = 5^2 + 12^2 - 2(5)(12)\cos B$

$64 = 25 + 144 - 120\cos B$

$= 169 - 120\cos B$

$0.875 = \cos B$

$29° \approx B$

$c^2 = a^2 + b^2 - 2ab\cos C$

$12^2 = 5^2 + 8^2 - 2(5)(8)\cos C$

$144 = 25 + 64 - 80\cos C$

$144 = 89 - 80\cos C$

$-89 \quad -89$

$55 = -80\cos C$

$-80 \qquad -80$

$-0.6875 = \cos C$

$133.4° \approx C$

$A \approx 17.6°$

Example

Solve ΔABC, where $\angle A = 46.5°$, $b = 10.5$, and $c = 18$

Heron's (Area) Formula

$s(a+b)$

An interesting application of the Law of Cosines involves a formula for finding the area of a triangle from the lengths of its three sides. We won't derive the formula here for time's sake. (see textbook)

<u>Heron's Formula</u>- For ΔABC the area $\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$, which is the **<u>semiperimeter</u>** (half perimeter).

Ex. Find the area of a triangle with give side lengths:
$a = 280$, $b = 125$, and $c = 315$

$$s = \frac{280+125+315}{2} = 360$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

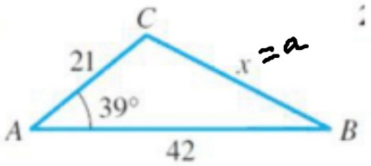
$$= \sqrt{360(360-280)(360-125)(360-315)}$$

$$\approx 17,451.6$$

Homework Problems

Use the Law of Cosines to determine the indicated side x or angle θ.

1.

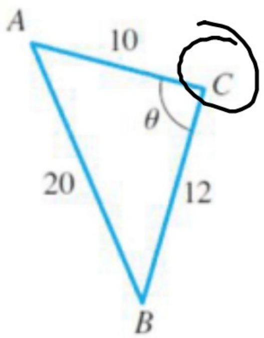


$a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 21^2 + 42^2 - 2(21)(42)\cos 39$

$x^2 \approx 834.1 \Rightarrow \boxed{x \approx 28.9}$

8.



$c^2 = a^2 + b^2 - 2ab\cos C$

Homework Problems

Solve the triangle.

11. a =3, $b = 4$, $\angle C = 53°$

Homework Problems

Solve the triangle.

17. $a = 50, b = 65, \angle A = 55°$

Homework Problems

Find the area of the triangle.

27. $a = 9, b = 12, c = 15$

Homework 1/29

TB pg. 513 #1, 3, 5, 8, 11-17 (odd), 27, 29