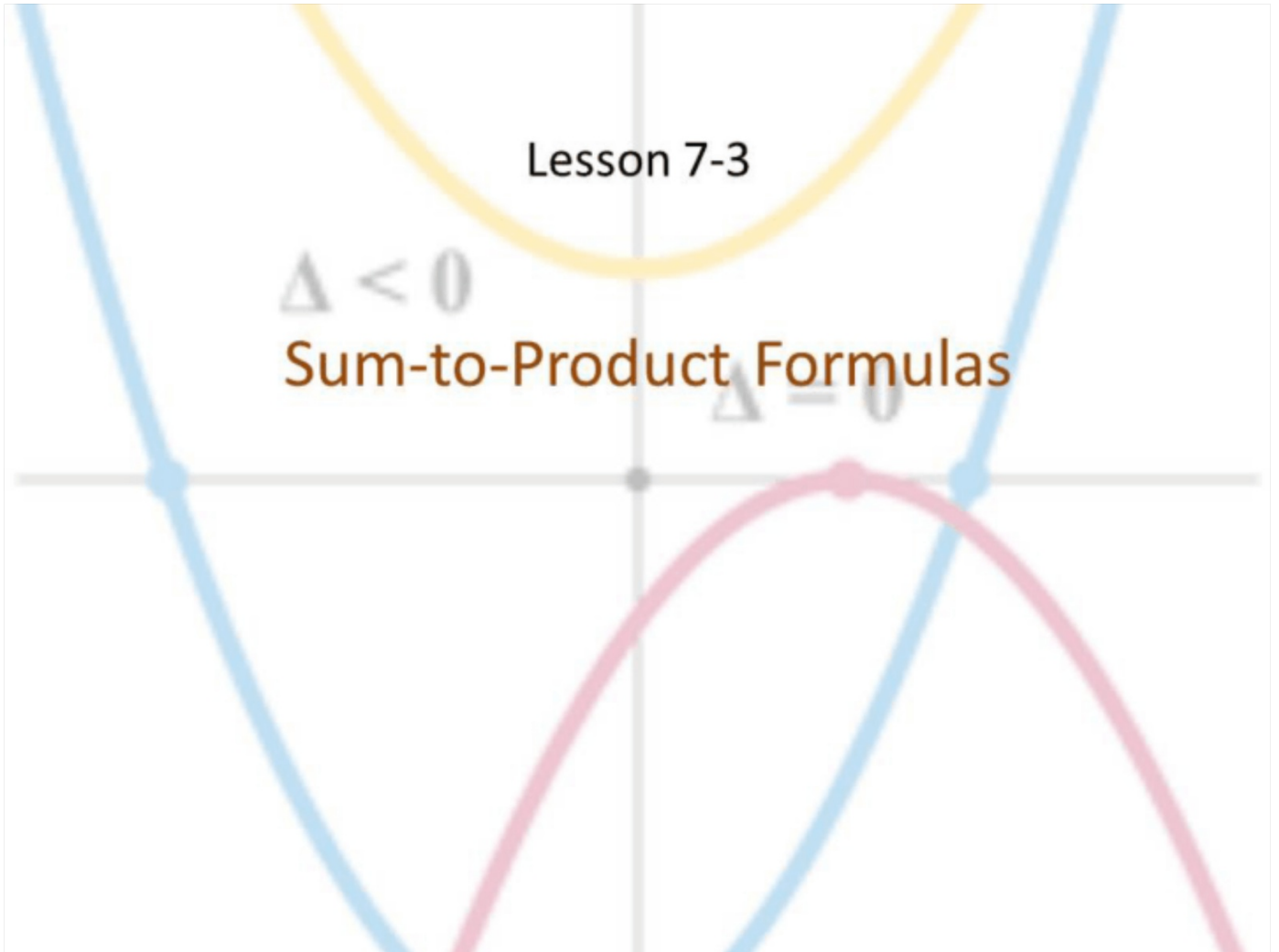


Lesson 7-3

$\Delta < 0$

Sum-to-Product Formulas

$\Delta = 0$



Objective

Students will...

- Be able to know the Sum-to-Product Formulas.
- Be able to use the Sum-to-Product formulas to prove identities.

Sum-to-Product Formulas

We now move further into different formulas for trig functions. The following is the **Sum-to-Product** formulas, which do exactly as it says—turn sums (addition or subtraction) into products (multiplication).

Sum-to-Product Formulas:

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = 2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

Using Sum-to-Product Formulas

Write $\sin 7x + \sin 3x$ as a product.

$$= 2 \sin \frac{7x+3x}{2} \cos \frac{7x-3x}{2}$$
$$= \boxed{2 \sin 5x \cos 2x}$$

Write $\sin 11x + \sin 5x$ as a product.

$$= \boxed{2 \sin 8x \cos 3x}$$

$$\sin(-x) = -\sin x.$$

Homework Problem

Write the sum as a product.

$$49. \cos 4x - \cos 6x$$

$$= 2 \sin \frac{4x+6x}{2} \sin \frac{4x-6x}{2}$$

$$= 2 \sin 5x \sin(-x)$$

$$= \boxed{-2 \sin 5x \sin x}$$

Using Sum-to-Product Formulas

Verify the identity: $\frac{\sin 3x - \sin x}{\cos 3x + \cos x} = \tan x$

$$\Rightarrow \frac{\cancel{2} \cos \frac{3x+x}{2} \sin \frac{3x-x}{2}}{\cancel{2} \cos \frac{3x+x}{2} \cos \frac{3x-x}{2}} = \tan x$$

$$\Rightarrow \frac{\sin x}{\cos x} = \tan x$$

$$\Rightarrow \tan x = \tan x \quad \checkmark$$

Using Sum-to-Product Formulas

Verify the identity: $\frac{\sin 4x + \sin 2x}{\sin 2x} = \frac{\sin 3x}{\sin x}$

Homework Problems

Verify the identity:

~~71. $\frac{\sin x + \sin 5x}{\cos x + \cos 5x} = \tan 3x$~~

$$72. \frac{\sin 3x + \sin 7x}{\cos 3x - \cos 7x} = \cot 2x$$

$$\Rightarrow \frac{2 \sin \frac{3x+7x}{2} \cos \frac{3x-7x}{2}}{2 \sin \frac{3x+7x}{2} \sin \frac{3x-7x}{2}} = -\cot 2x$$

$$\Rightarrow \frac{\cos(-2x)}{\sin(-2x)} = -\cot 2x$$

$$\Rightarrow \cot(-2x) = -\cot 2x$$

$$\Rightarrow -\cot 2x = -\cot 2x \quad \checkmark$$

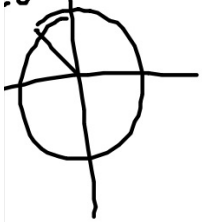
Homework Problems

~~79. Show that $\sin 45^\circ + \sin 15^\circ = \sin 75^\circ$~~

77. Show that $\sin 130^\circ - \sin 110^\circ = -\sin 10^\circ$

$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$ ANSWER.

$\frac{\pi}{3}$
 20° | 90°



$$\sin 130^\circ - \sin 110^\circ = 2 \cos \frac{130^\circ + 110^\circ}{2} \sin \frac{130^\circ - 110^\circ}{2}$$

$$= 2 \cos 120^\circ \sin 10^\circ$$

$$= 2 \left(-\frac{1}{2}\right) \sin 10^\circ$$

$$= \boxed{-\sin 10^\circ}$$

Homework 2/17

TB pg. 548-549 #47-53 (odd), 72, 74, 75, 77