Warm Up 2/11

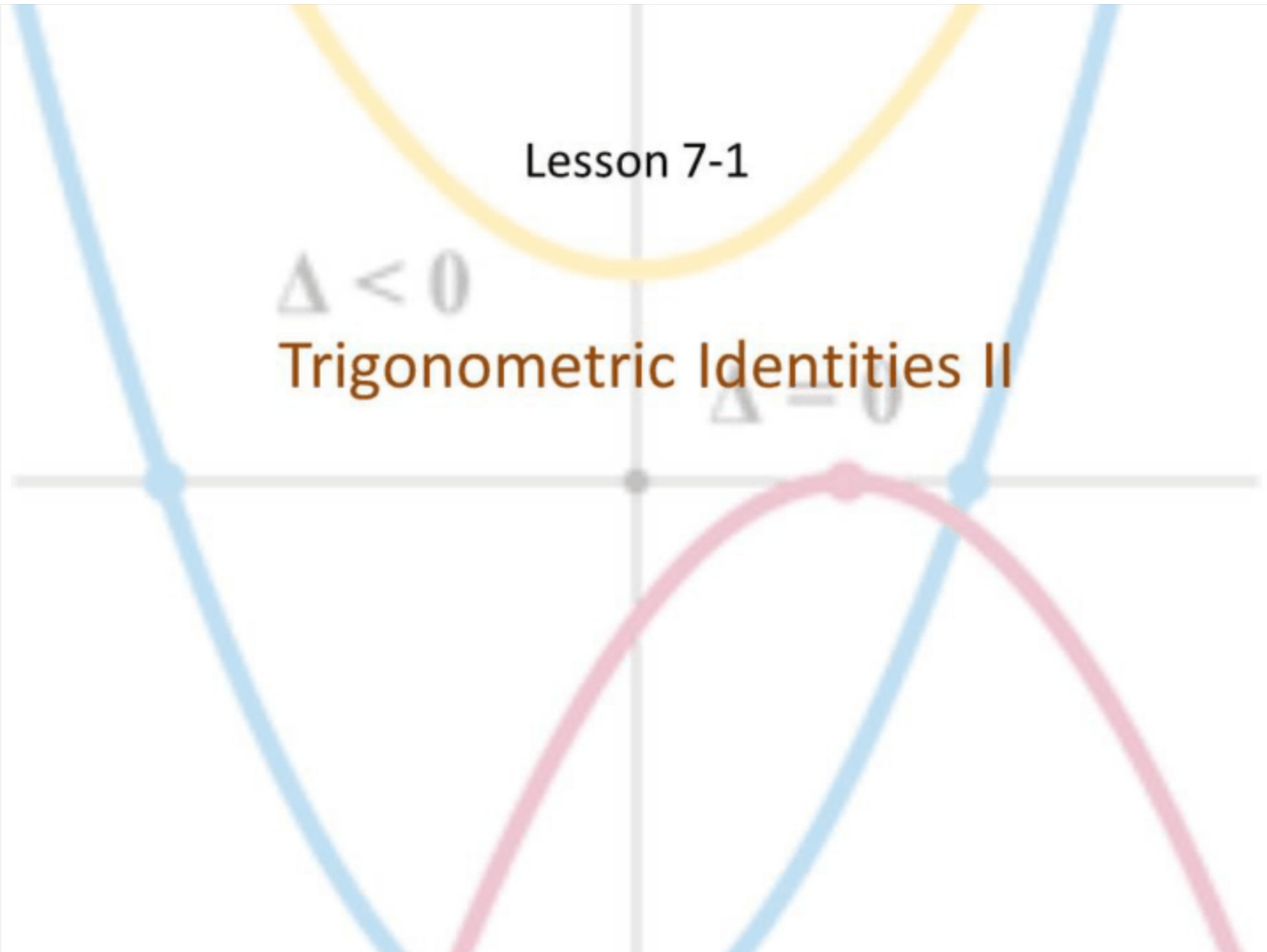
Simplify the trigonometric expression.

1. $\frac{\sec x - \cos x}{\tan x} = \frac{\frac{1}{\cos x} - \frac{\cos x \cdot \cos x}{1 \cdot \cos x}}{\frac{\sin x}{\cos x}} = \frac{\frac{1 - \cos^2 x}{\cos x}}{\frac{\sin x}{\cos x}}$

$$= \frac{\sin^{\cancel{2}} x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{\cancel{\sin x}} = \boxed{\sin x}$$

Lesson 7-1

Trigonometric Identities II



Objective

Students will…

- Be able to prove or verify trigonometric identities.
- ~~Be able to simplify expressions using trigonometric substitution.~~

Trigonometric Identities

Before we get any deep into trig analysis, we must first recall some of the basic trigonometric identities and definitions. Primarily,

$\csc x = \frac{1}{\sin x}$ $\qquad$ $\sec x = \frac{1}{\cos x}$ $\qquad$ $\cot x = \frac{1}{\tan x}$

$\tan x = \frac{\sin x}{\cos x}$ $\qquad$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$

From this, we also get:

$\sin^2 x = 1 - \cos^2 x$ and $\cos^2 x = 1 - \sin^2 x$

$\tan^2 x + 1 = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$

$\tan^2 x = \sec^2 x - 1$ and $\cot^2 x = \csc^2 x - 1$

Methods for Proving Identities

One of the main components of trig analysis is to prove identities. There are two different methods for proving identities:

I. Rewrite **one** of the sides to match the other side.

Ex.

$$x + 3 = 6\left(\frac{1}{\cancel{6}}x + \frac{11}{\cancel{6}} - \frac{13}{\cancel{6}} + \frac{5}{\cancel{6}}\right)$$

$$= x + 11 - 13 + 5$$

$$x + 3 = x + 3 \checkmark$$

II. Modify **both** sides until they are the same.

Ex.

$$3(2x - 1) = 2x + \cancel{2}\left(2x - \frac{3}{\cancel{2}}\right)$$

$$= 2x + 4x - 3$$

$$6x - 3 = 6x - 3 \checkmark$$

Guidelines for Proving Identities

Furthermore, we have some guidelines/tips for proving identities.

1. **Focus on the fractions**: More often than not, identity proofs are more easily done when you work with the side that involves a fraction.

2. **Pick the more "complicated" side**: It's easier to modify the sides that has less sines or cosines. Generally, rewriting everything as sine or cosine can help you when you are "stuck."

3. **Use the Known Identities!**: Use algebra and the identities are already known to you. Look to combine multiple fractions into one with a common denominator.

Example

Prove/Verify the identity: $\cos\theta\,(\sec\theta - \cos\theta) \stackrel{?}{=} \sin^2\theta$

$\Rightarrow \cos\theta\sec\theta - \cos^2\theta = \sin^2\theta$

$\Rightarrow \cancel{\cos\theta}\left(\cancel{\frac{1}{\cos\theta}}\right) - \cos^2\theta = \sin^2\theta$

① $\Rightarrow 1 - \cos^2\theta = \sin^2\theta$

$\Rightarrow \sin^2\theta = \sin^2\theta$ ✓

② $1 - \cos^2\theta = \sin^2\theta$

$1 - \cos^2\theta = 1 - \cos^2\theta$ ✓

Example

Prove/Verify the identity: $\cos x \tan x = \sin x$

$$\Rightarrow \cancel{\cos x}\,\frac{\sin x}{\cancel{\cos x}} = \sin x$$

$$\Rightarrow \sin x = \sin x \checkmark$$

Homework Problems

Verify the identity:

31. $\sin B + \cos B \cot B = \csc B$

$$\Rightarrow \sin B + \cos B \frac{\cos B}{\sin B} = \csc B$$

$$\Rightarrow \frac{\sin B}{\sin B} \cdot \frac{\sin B}{1} + \frac{\cos^2 B}{\sin B} = \csc B$$

$$\Rightarrow \frac{\sin^2 B + \cos^2 B}{\sin B} = \csc B$$

$$\Rightarrow \frac{1}{\sin B} = \csc B$$

$$\Rightarrow \csc B = \csc B \checkmark$$

Homework Problems

Verify the identity :

29. $\frac{\tan y}{\csc y} = \sec y - \cos y$

$\Rightarrow \dfrac{\frac{\sin y}{\cos y}}{\frac{1}{\sin y}} = \sec y - \cos y$

$\Rightarrow \frac{\sin y}{\cos y} \cdot \frac{\sin y}{1} = \sec y - \cos y$

$\Rightarrow \frac{\sin^2 y}{\cos y} = \sec y - \cos y$

$\frac{1}{a} - \frac{2}{a} = \frac{1-2}{a}$

$\Rightarrow \frac{\sin^2 y}{\cos y} = \frac{1}{\cos y} - \frac{\cos y \cdot \cos y}{1 \cdot \cos y}$

$\Rightarrow \frac{\sin^2 y}{\cos y} = \frac{1-\cos^2 y}{\cos y}$

$\Rightarrow \frac{\sin^2 y}{\cos y} = \frac{\sin^2 y}{\cos y}$ ✓

$\Rightarrow \frac{1-\cos^2 y}{\cos y} = \sec y - \cos y$

$\Rightarrow \frac{1}{\cos y} - \frac{\cos^{\cancel{2}} y}{\cancel{\cos y}} = \sec y - \cos y$

$= \sec y - \cos y = \sec y - \cos y$

Homework Problems

Verify the identity :

35. $\tan\theta + \cot\theta = \sec\theta\csc\theta$

$$\Rightarrow \frac{\sin\theta \cdot \sin\theta}{\cos\theta \cdot \sin\theta} + \frac{\cos\theta \cdot \cos\theta}{\sin\theta \cdot \cos\theta} = \sec\theta\csc\theta$$

$$\Rightarrow \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta} = \sec\theta\csc\theta$$

$$\Rightarrow \frac{1}{\cos\theta\sin\theta} = \sec\theta\csc\theta$$

$$\Rightarrow \sec\theta\csc\theta = \sec\theta\csc\theta \checkmark$$

Homework Problems

$\frac{a}{1+i}\cdot\frac{1-i}{1-i}$

Verify the identity :

51. $\frac{1-\cos\alpha}{\sin\alpha}=\frac{\sin\alpha}{1+\cos\alpha}$

$\Rightarrow \frac{1}{\sin a}-\frac{\cos a}{\sin a}=\frac{\sin a}{1+\cos a}$

$\Rightarrow \csc a-\cot a=\frac{\sin a}{1+\cos a}$

$\frac{1-\cos a}{\sin a}=\frac{\sin a}{1+\cos a}\cdot\frac{1-\cos a}{1-\cos a}$

$\frac{1-\cos a}{\sin a}=\frac{\sin a-\sin a\cos a}{1-\cos^2 a}$

Math

Homework Problems

Verify the identity :

85. $\frac{1+\sin x}{1-\sin x} = (\tan x + \sec x)^2$

Homework 2/11

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