Warm Up 12/5

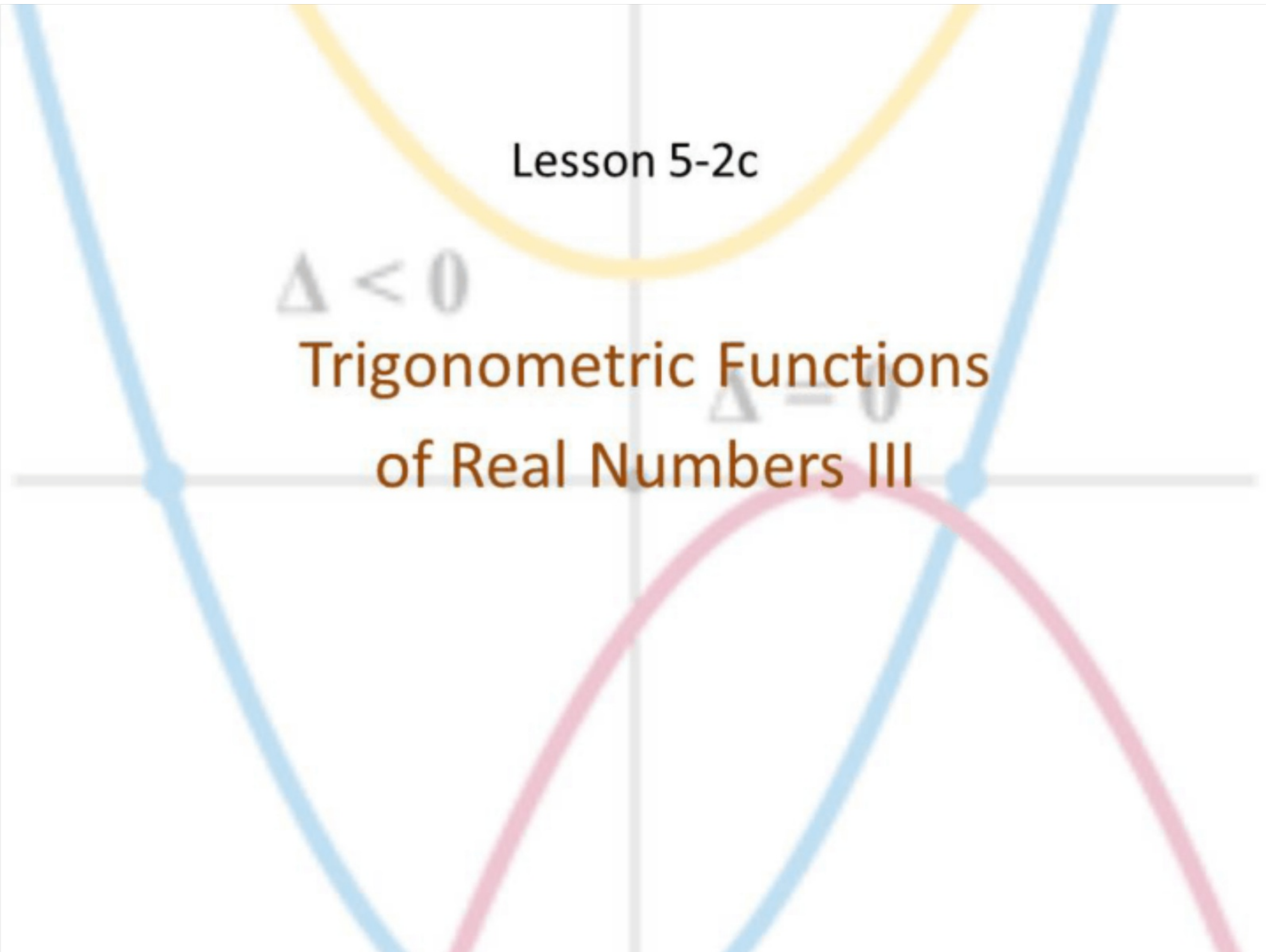
Evaluate the following trig functions without using a calculator.

1. $\sin\frac{2\pi}{3}$

2. $\cos\frac{11\pi}{6}$

3. $\sec\frac{\pi}{4}$

4. $\tan\frac{4\pi}{3}$


Lesson 5-2c
Δ < 0
Trigonometric Functions
of Real Numbers III

Objective

Students will…

- Be able to rewrite trigonometric functions using the fundamental identities.
- ~~Find all trigonometric functions from the value of one using the fundamental identities.~~

Soh Cah Toa

Recall that given a right triangle…

$$\sin t = \frac{opposite}{hypotenuse} \qquad \cos t = \frac{adjacent}{hypotenuse} \qquad \tan t = \frac{opposite}{adjacent}$$

$$\csc t = \frac{1}{\sin t} = \frac{hypotenuse}{opposite} \qquad \sec t = \frac{1}{\cos t} = \frac{hypotenuse}{adjacent}$$

$$\cot t = \frac{1}{\tan t} = \frac{adjacent}{opposite}$$

We can use the properties of right triangles to figure out the rest of the trigonometric functions.

Soh Cah Toa.

$\sin t = -\frac{4}{5} = \frac{opp}{hyp}$ $\cos t = \frac{3}{5} = \frac{adj.}{hyp}$ $\tan t = -\frac{4}{3}$

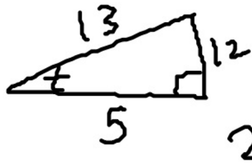
$\csc t = -\frac{5}{4}$ $\sec t = \frac{5}{3}$

$\cot t = -3/4$

$\frac{adj.}{hyp.}$

If $\cos t = -\frac{5}{13}$, and t is in quadrant II, find the values of all the trigonometric functions at t.

Soh Cah Toa.



$a^2+b^2=c^2$

$5^2+b^2=13^2$

$25+b^2=169$

$b^2=144$

$b=12$

$\sin t = \frac{12}{13}$

$\tan t = -\frac{12}{5}$

$\csc t = \frac{13}{12}$

$\sec t = -\frac{13}{5}$

$\cot t = -\frac{5}{12}$

Trigonometric Functions

Recall from the definitions of trigonometric functions that…

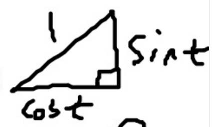
$$\tan t = \frac{\sin t}{\cos t} \qquad \cot t = \frac{1}{\tan t} = \frac{\cos t}{\sin t}$$

$$\sec t = \frac{1}{\cos t} \qquad \csc t = \frac{1}{\sin t}$$

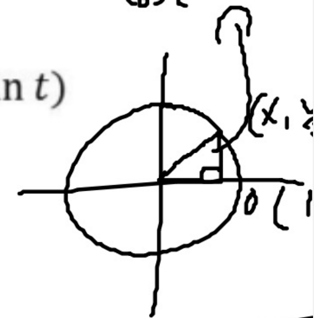
$\cos t^2 \neq (\cos t)^2$

Coordinates on a Unit Circle

Now, also recall that on the unit circle, we defined the following:



$\cos t = x$ $\quad\quad$ $\sin t = y$ $\quad\quad$ $\rightarrow$ $\quad\quad$ $(x, y) = (\cos t\,, \sin t)$



Now, let's see how this can be applied on a unit circle.

$$a^2 + b^2 = c^2$$

$$= (\cos t)^2 + (\sin t)^2 = 1^2$$

$$= \boxed{\cos^2 t + \sin^2 t = 1}$$

Notation:

$(\cos t)^2 = \cos^2 t$

$(\sin t)^2 = \sin^2 t$

Pythagorean Identities

Hence, we can now conclude the following identities:

Pythagorean Identities: (Note: $sin^2 t = (\sin t)^2$)

$sin^2 t + cos^2 t = 1$ $\qquad$ $tan^2 t + 1 = sec^2 t$ $\qquad$ $1 + cot^2 t = csc^2 t$

Also, moving some of these around using algebra:

$\sin t = \pm\sqrt{1 - cos^2 t}$ $\qquad$ $\cos t = \pm\sqrt{1 - sin^2 t}$

$\Rightarrow \sin^2 t = 1 - \cos^2 t$ $\qquad$ $\Rightarrow \cos^2 t = 1 - \sin^2 t$

Rewriting Trigonometric Functions

We can also rewrite trigonometric functions using others.

Example: Write $\tan t$ in terms of $\cos t$, where t is in quadrant III.

$$\tan t = \frac{\sin t}{\cos t} = \frac{\pm\sqrt{1-\cos^2 t}}{\cos t}$$

$$\boxed{= -\frac{\sqrt{1-\cos^2 t}}{\cos t}}$$

Examples

$$\cos t = \pm\sqrt{1-\sin^2 t}$$

Write $\tan t$ in terms of $\sin t$, where t is in quadrant I.

$$\tan t = \frac{\sin t}{\cos t} = \boxed{\frac{\sin t}{\pm\sqrt{1-\sin^2 t}}} \cdot \frac{\sqrt{1-\sin^2 t}}{\sqrt{1-\sin^2 t}}$$

$$= \frac{\sin t\sqrt{1-\sin^2 t}}{1-\sin^2 t}$$

Write $\sec t$ in terms of $\tan t$, where t is in quadrant II

$$\sqrt{\sec^2 t} = \sqrt{1+\tan^2 t}$$

$$\sec t = -\sqrt{1+\tan^2 t}$$

Homework 12/5

TB pg. 417 #53-61 (odd), 63, 64

Application of Pythagorean Identities

Although these identities will be used much more extensively in the future, we can still make use of them here.

Example: If $\cos t = \frac{3}{5}$ and t is in quadrant IV, find the values of all other trigonometric functions at t.

We need to first find $\sin t$. We use our identity: $\sin t = \pm\sqrt{1 - cos^2 t}$

$$\sin t = \pm\sqrt{1 - cos^2 t} = \pm\sqrt{1 - \left(\frac{3}{5}\right)^2} = \pm\sqrt{1 - \left(\frac{9}{25}\right)} = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$$

However, since we are told that t is quadrant IV, $\sin t = -\frac{4}{5}$