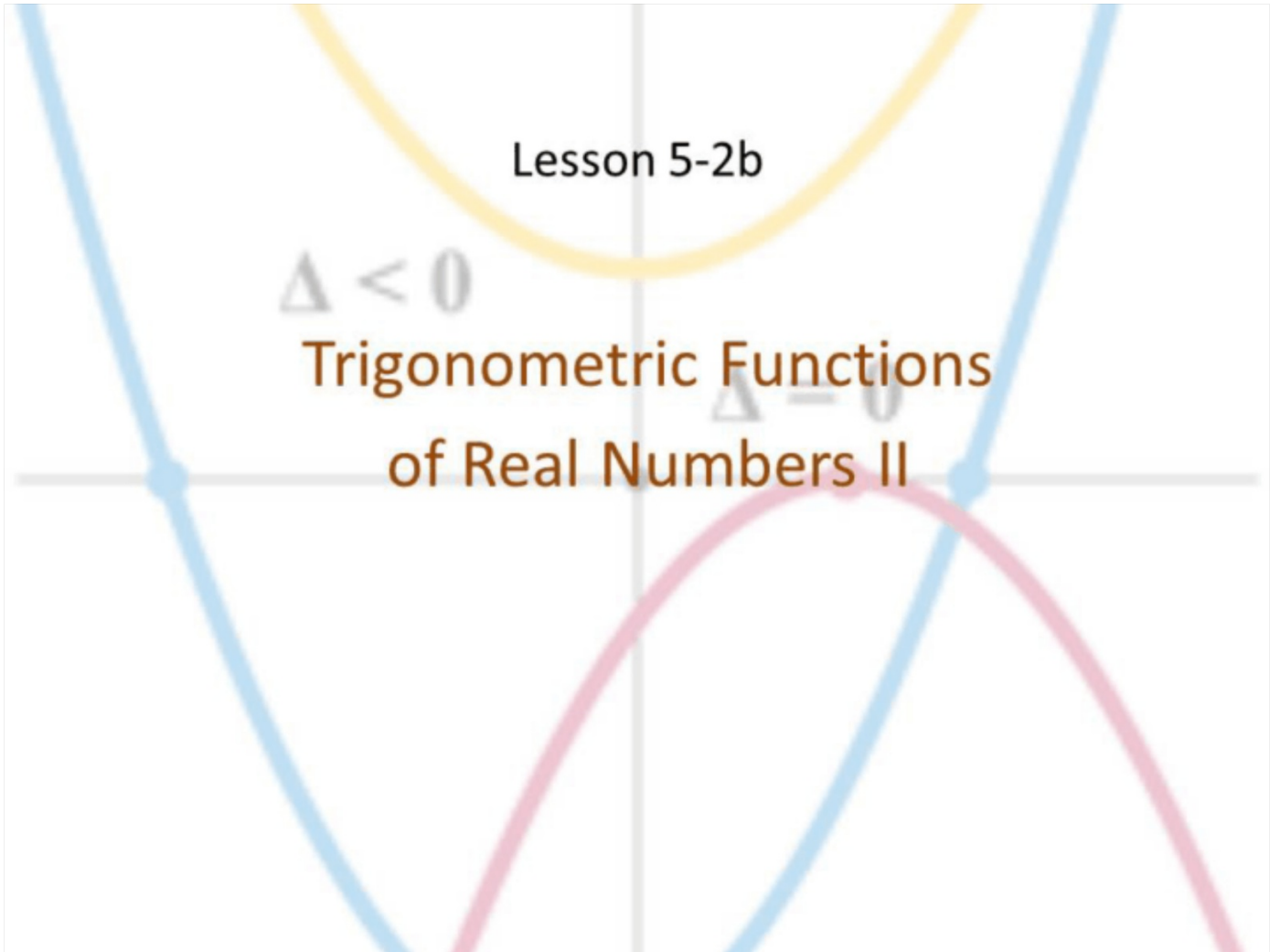Lesson 5-2b

Trigonometric Functions
of Real Numbers II

Objective

Students will...

- Be able to use a calculator to evaluate trigonometric functions (both radians and degree)
- Be able to know and apply the even-odd properties of trigonometric functions.

Trigonometric Functions

The concept of trigonometric functions can be defined in terms of the unit circle. The **definition of trigonometric functions** is as follows:

$$\cos t = x \qquad\qquad \sin t = y \qquad\qquad \tan t = \frac{y}{x} \;(x \neq 0) \quad \frac{\sin t}{\cos t}$$

$$\sec t = \frac{1}{\cos t} = \frac{1}{x} \qquad \csc t = \frac{1}{\sin t} = \frac{1}{y} \qquad \cot t = \frac{1}{\tan t} = \frac{x}{y} \quad \frac{\cos t}{\sin t}$$

$$(x \neq 0) \qquad\qquad\qquad (y \neq 0) \qquad\qquad\qquad (y \neq 0)$$

Evaluating Trigonometric Functions

The previous definitions of trigonometric functions are only helpful to us if t happens to be one of the values on the unit circle. Consider for example, $\cos \dfrac{\pi}{7}$

Since $\dfrac{\pi}{7}$ is not on the unit circle, we would have to use a calculator to evaluate this function.

One thing to keep in mind is that trigonometric functions can be evaluated using both degrees and radians. It is **CRUCIAL** that the calculator is in the right mode. For most calculators, DEG=Degrees, and RAD=Radians.

So, going back to our problem, since $\frac{\pi}{7}$ is a value in radians (no degree sign), we need to put our calculator in radian mode.

$$\cos\frac{\pi}{7} \approx 0.9\,00977$$

Try doing these problems. Note that they are all in radians.

$$\sin\frac{\pi}{5} \approx 0.58778 \qquad \tan\frac{7\pi}{9} \approx -0.8391 \qquad \cos\frac{6\pi}{31} \approx 0.82208$$

$$\tan\frac{11\pi}{8} \approx 2.4142 \qquad\qquad\qquad \sin\frac{7\pi}{5} \approx -0.9511$$

Now, let's try a couple problems in degree mode. $\sin^{-1} \neq \csc$

$\sin 33° \approx 0.5446$ $\cos 67° \approx 0.3907$ $\tan 0.889° \approx 0.0155$

Also, remember that $\sec t = \dfrac{1}{\cos t}$, $\csc t = \dfrac{1}{\sin t}$, and $\cot t = \dfrac{1}{\tan t}$

Thus, $\csc 67° = \dfrac{1}{\sin 67} \approx 1.0864$ and $\sec \dfrac{\pi}{8} = \dfrac{1}{\cos \pi/8} \approx 1.0824$
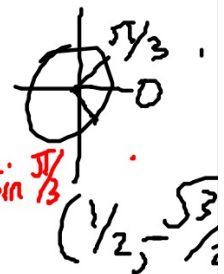
Try these. Make sure you're in the right mode.

$\cot \dfrac{\pi}{19} \approx 5.9927$ $\csc 65.98° \approx 1.0948$ $\sec \dfrac{27\pi}{16} \approx 1.79995$.

$\pi/_3 \rightarrow (1/_2, \sqrt{3}/_2)$

Even-Odd Properties

Consider the following.

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \sin\left(-\frac{\pi}{3}\right) = \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2} = -\sin\frac{\pi}{3}$$

$(1/_2, \sqrt{3}/_2)$

Now, what about…

$$\cos \frac{\pi}{3} = 1/_2 \qquad \cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{5\pi}{3}\right) = 1/_2$$

$(1/_2, -\sqrt{3}/_2)$

Turns out, these results can be generalized.

$$\tan t = \frac{\sin t}{\cos t}$$

$$\Rightarrow \tan(-t) = \frac{\sin(-t)}{\cos(-t)} = \left(\frac{\sin t}{\cos t}\right)$$

Even-Odd Properties:

$$\cos(-t) = \cos t \qquad \sin(-t) = -\sin t \qquad \tan(-t) = -\tan t$$

$$\csc(-t) = -\csc(t) \qquad \sec(-t) = \sec t \qquad \cot(-t) = -\cot t$$

$\pi/6 \rightarrow \left(\frac{\sqrt{3}}{2}, \frac{1}{2} \right)$

Examples

Use the Even-Odd Properties to evaluate the following.

$\sin \left(\ominus \frac{\pi}{6} \right) = -\sin \frac{\pi}{6}$
$= -\left(\frac{1}{2} \right) = \boxed{-\frac{1}{2}}$

$\cos \left(-\frac{\pi}{4} \right) = \cos \frac{\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$

$\cot \left(\ominus \frac{5\pi}{6} \right) = -\cot \left(\frac{5\pi}{6} \right)$

$\csc \left(\ominus \frac{2\pi}{3} \right) = -\csc \left(\frac{2\pi}{3} \right)$

$\tan \left(\ominus \frac{11\pi}{6} \right) = -\tan \left(\frac{11\pi}{6} \right)$

$\sec \left(-\frac{\pi}{2} \right) = \sec \left(\frac{\pi}{2} \right)$

$\cos(-t) = \cos t$

$-\cos t \neq \cos t$

Homework 12/3

Worksheet