Warm Up 12/10

$2 \div 1/2$

$0.6 = 6/10 =$

Find the period and the amplitude of the following.

1. $2\sin 4\pi t$

Amp: $|2| = 2$

Per: $2\cancel{\pi}/4\cancel{\pi} = 1/2$

2. $5\cos\frac{1}{2}\pi t$

Amp: $|5| = 5$

Per: $2\cancel{\pi}/\tfrac{1}{2}\cancel{\pi} = 4$

3. $7\sin 0.6\pi t$

Amp: $|7| = 7$

Per: $2\cancel{\pi}/0.6\cancel{\pi} = 2/\tfrac{3}{5} = \boxed{10/3}$

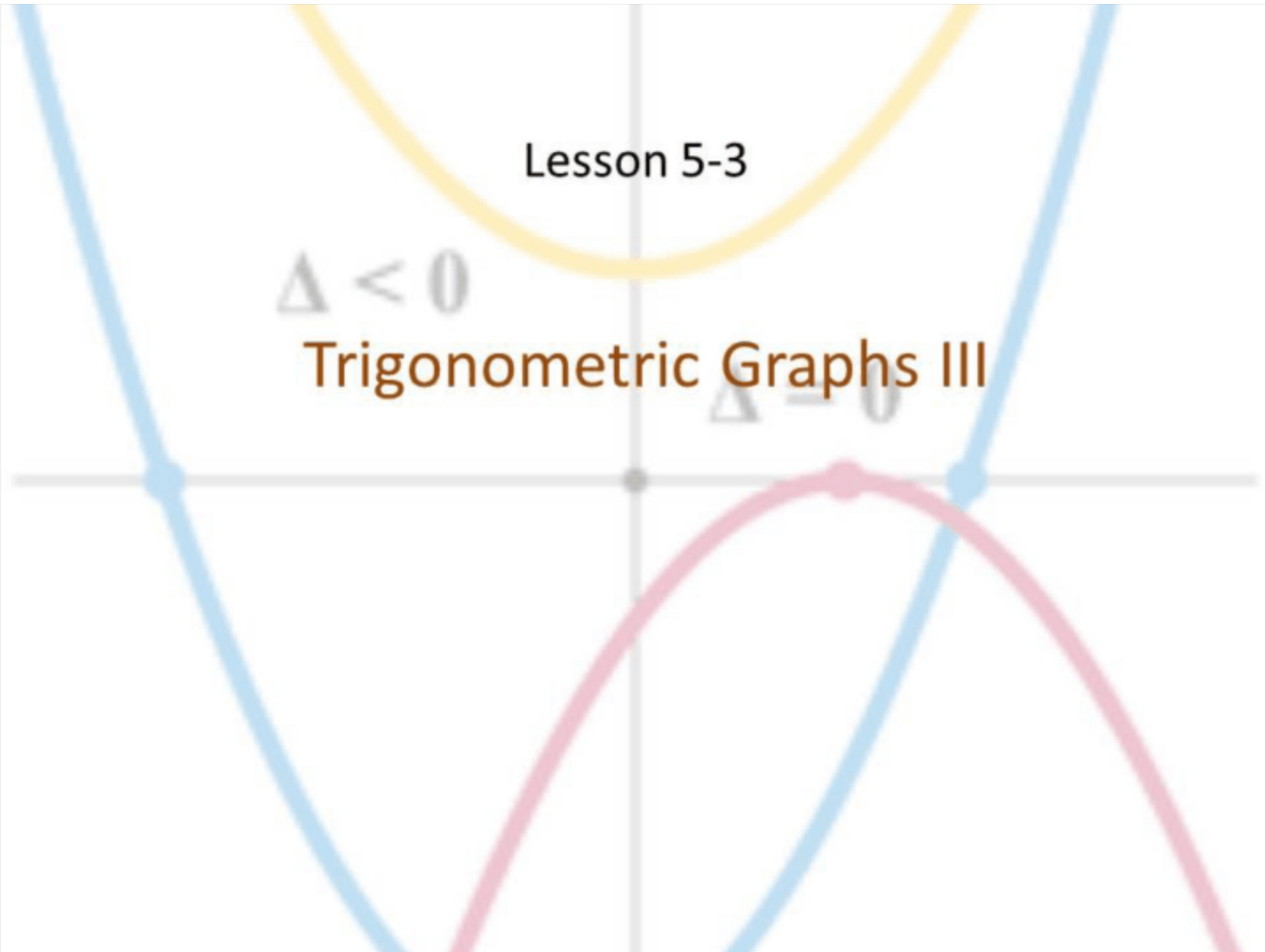
4. What does it mean for a function to have a period of 3π?

The length of each repetition (cycle) is 3π.

$2\pi/k = 3\pi$

Lesson 5-3

Trigonometric Graphs III



Objective

Students will…

- Be able to identify and graph the shift of sine and cosine functions.

Standard Equation of Sine and Cosine Curves

Like any other functions, there exists a standard equation of both sine and cosine curves.

Sine Curves: Any equation of a sine curve is written in the form:

$y = a \sin kx$, where a and k are real numbers with $k > 0$

Cosine Curves: Any equation of a cosine curve is written in the form:

$y = a \cos kx$, where a and k are real numbers with $k > 0$

Period and Amplitude of Sine and Cosine Curves

In our previous lesson we simply used the graph to figure out the period and amplitude of a given sine or cosine curve. However, we may not (more of than not) have a graph to refer to. In fact, how would we find the period if we were asked to graph a given sine or cosine curve? Of course, we can use the x-y table to graph the curve first, but this isn't always practical.

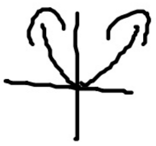
Fortunately, finding the period and the amplitude of a sine or cosine curve can be found algebraically from their equation.

For sine and cosine curves: $y = a \sin kx$ and $y = a \cos kx$,

Period = $\frac{2\pi}{k}$ **Amplitude** = $|a|$

Horizontal and Vertical Shift

Recall from chapter 2 about the shift of parabolas. The standard equation of a parabola is $y = x^2$. Now, consider…



Ex.

$y = ax^2 + bx + c$

$y = x^2$ $\qquad\qquad$ $y = (x-4)^2 - 9$

Vertex: $(0,0) \xrightarrow{(0+4,\,0-9)} (4,-9)$

right 4, down 9.

Shift: None

Horizontal and Vertical Shift

$\frac{4\pi}{2}$ $\frac{2\pi}{2}$ $\pi + \pi/2$

Believe it or not, trig functions (along with many other functions) take the similar format when it comes to their shifts.

$a\cos kx$

Ex.

$$y = \cos x \qquad\qquad y = \cos\left(x - \frac{\pi}{2}\right) + 1$$

Period: $2\pi/1 = 2\pi$ | $2\pi/1 = 2\pi$

Amplitude: $|1| = 1$ | $|1| = 1$

Shift: None | $+\pi/2$, $+1$, right $\pi/2$, up 1.

High/Low

Start/End Point: (1 cycle)

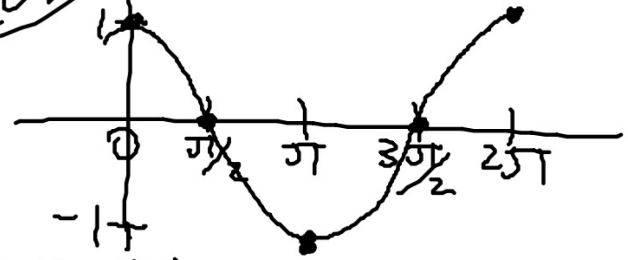
y-axis: -1 to 1 | y-axis: $-1+1$ to $1+1$, 0 to 2

x-axis: 0 to 2π | x-axis: $0+\pi/2$ to $2\pi+\pi/2$, $\pi/2$ to $5\pi/2$

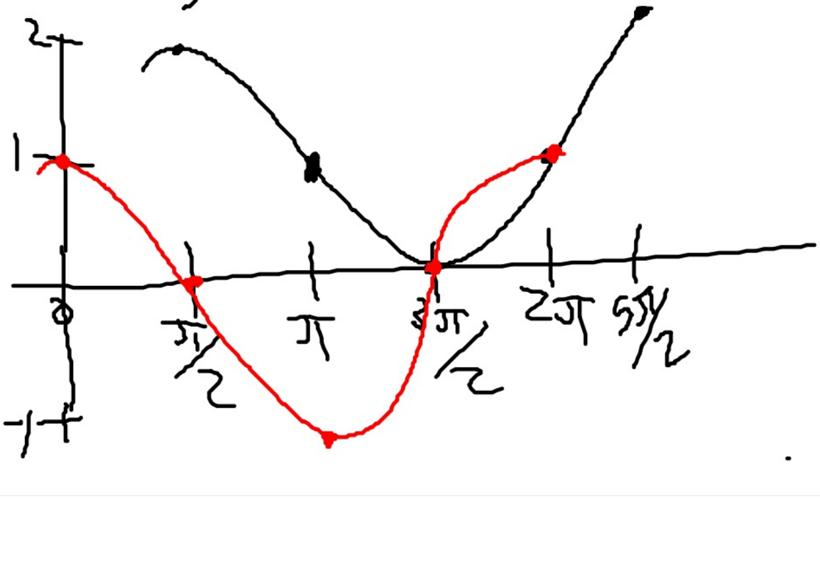
Examples

cos x

Let's graph the two and compare. $y = \cos x$, $y = \cos\left(x - \frac{\pi}{2}\right) + 1$



$= \cos(x - \pi/2) + 1$



Example

Believe it or not, trig functions (along with many other functions) take the similar format when it comes to their shifts.

Ex.

$y = 3\sin(2x - \pi/2)$

$$y = \sin x \qquad\qquad y = 3\sin 2(x - \tfrac{\pi}{4}) + 0$$

Period: 2π $\qquad 2\pi/2 = \pi$

Amplitude: $|1| = 1$ $\qquad |3| = 3$

Shift: None $\qquad$ right $\pi/4$

Start/End Point:

y-axis: -1 to 1 $\qquad$ y-axis: -3 to 3

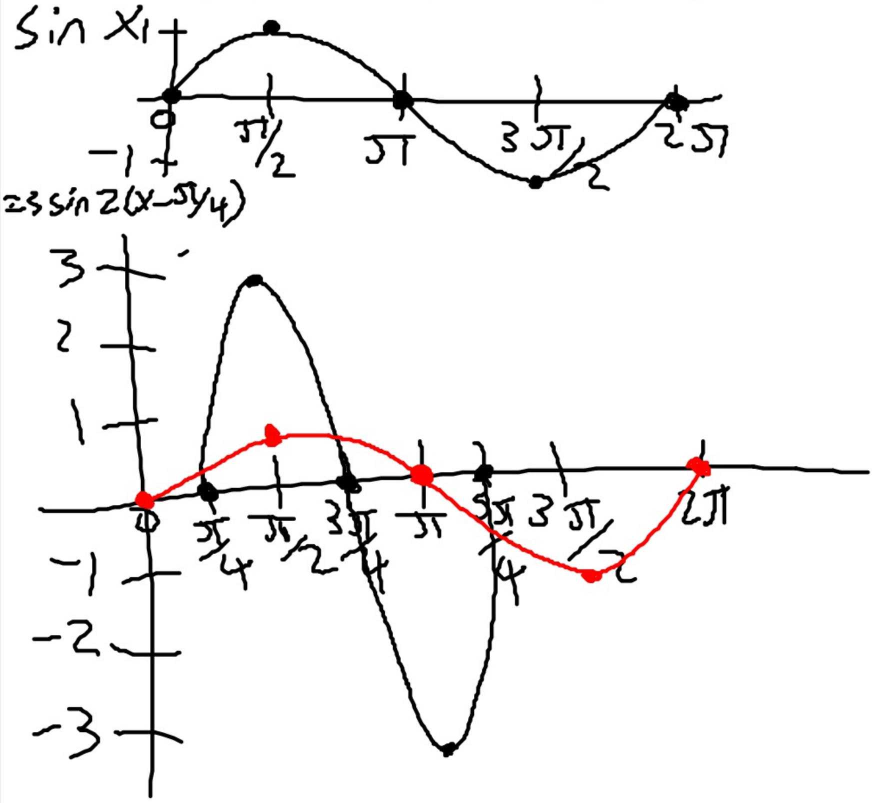
x-axis: 0 to 2π $\qquad$ x-axis: $0 + \pi/4$ $\quad \pi + \pi/4$

$\pi/4$ to $5\pi/4$

Examples

Let's graph the two and compare. $y = \sin x$, $y = 3\sin 2\left(x - \frac{\pi}{4}\right)$

$\pi/4$ to $5\pi/4$.



Guidelines to Graphing

1. Identify whether it is a sine or a cosine function.
2. Find the period and the amplitude.
3. Find the phase shift of the functions.
4. Identify the starting point and the endpoint of the shifted graph.
5. Graph

Examples

Graph the following (pg. 429)

1. $f(x) = 1 + \cos x$

Examples

Graph the following (pg. 429)

33. $y = 5\cos(3x - \frac{\pi}{4})$

Homework 12/10

TB pg. 429 #1, 11, 19, 27, 33, 36

(**Be sure to graph!)**