

$$1. \frac{\tan x}{1} + \frac{\cos x}{1 + \sin x} = \frac{1}{\cos x}$$

$$\Rightarrow \frac{\tan x + \tan x \sin x + \cos x}{1 + \sin x} = "$$

$$\Rightarrow \frac{\frac{\sin x}{\cos x} + \frac{\sin^2 x}{\cos x} + \frac{\cos x}{1}}{1 + \sin x} = "$$

$$\Rightarrow \frac{\frac{\sin x + \sin^2 x + \cos^2 x}{\cos x}}{1 + \sin x} = "$$

$$\Rightarrow \frac{\frac{\sin x + 1}{\cos x} \cdot \frac{1}{1 + \sin x}}{1 + \sin x} = "$$

$$\Rightarrow \frac{1}{\cos x} = \frac{1}{\cos x} \checkmark$$

$$4. \tan x + \csc x = \sec x \csc x$$

$$\Rightarrow \frac{\tan x}{1} + \frac{1}{\tan x} = \sec x \csc x$$

$$\Rightarrow \frac{\tan^2 x + 1}{\tan x} = "$$

$$\Rightarrow \frac{\sec^2 x}{\tan x} = "$$

$$\Rightarrow \frac{1}{\cos^2 x} \cdot \frac{\cos x}{\sin x} = "$$

$$\Rightarrow \frac{1}{\cos x \sin x} = "$$

$$\Rightarrow \sec x \csc x = \sec x \csc x \checkmark$$

$$2. \tan^2 x + 1 = \sec^2 x$$

$$\sec^2 x = \sec^2 x \checkmark$$

(id sheet)

$$3. \frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} = 2 \tan x \sec x$$

$$\Rightarrow \frac{1 + \sin x - (1 - \sin x)}{(1 - \sin x)(1 + \sin x)} = 2 \tan x \sec x$$

$$\Rightarrow \frac{2 \sin x}{1 - \sin^2 x} = 2 \tan x \sec x$$

$$\frac{2 \sin x}{1 - \sin^2 x} = 2 \tan x \sec x$$

$$\Rightarrow \frac{2 \sin x}{\cos^2 x} = 2 \tan x \sec x$$

$$\Rightarrow 2 \frac{\sin x}{\cos x \cos x} = 2 \tan x \sec x$$

$$\Rightarrow 2 \tan x \sec x = 2 \tan x \sec x \checkmark$$

5.

$$\frac{1 + \tan^2 x}{1 - \tan^2 x} = \frac{1}{\cos^2 x - \sin^2 x}$$

$$\Rightarrow \frac{\sec^2 x}{1 - \tan^2 x} = "$$

$$\Rightarrow \frac{\sec^2 x}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}} = "$$

$$\Rightarrow \frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\cos^2 x - \sin^2 x} = "$$

$$\Rightarrow \frac{1}{\cos^2 x - \sin^2 x} = \frac{1}{\cos^2 x - \sin^2 x} \checkmark$$

5.  $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$   
 $\Rightarrow \frac{\sin^2 x}{\cos^2 x} - \sin^2 x = \dots$   
 $\Rightarrow \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x} = \dots$   
 $\Rightarrow \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x} = \dots$   
 $\Rightarrow \frac{\sin^2 x (\sin^2 x)}{\cos^2 x} = \dots$   
 $\Rightarrow \tan^2 x \sin^2 x = \tan^2 x \sin^2 x$

7.  $\frac{1 - \cos x}{\sin x} + \frac{\sin x}{1 - \cos x} = 2 \csc x$   
 $\Rightarrow \frac{(1 - \cos x)^2 + \sin^2 x}{(\sin x)(1 - \cos x)} = 2 \csc x$   
 $\Rightarrow \frac{1 - 2\cos x + \cos^2 x + \sin^2 x}{\sin x (1 - \cos x)} = \dots$   
 $\Rightarrow \frac{2 - 2\cos x}{\sin x (1 - \cos x)} = \dots$   
 $\Rightarrow \frac{2(1 - \cos x)}{\sin x (1 - \cos x)} = \dots$   
 $\Rightarrow 2 \csc x = 2 \csc x \checkmark$

8.  $\frac{\sec x - 1}{\sec x + 1} = \frac{1 - \cos x}{1 + \cos x}$   
 $\Rightarrow \frac{\frac{1}{\cos x} - 1}{\frac{1}{\cos x} + 1} = \dots$   
 $\Rightarrow \frac{1 - \cos x}{1 + \cos x} = \dots$   
 $\Rightarrow \frac{1 - \cos x}{\cos x} \cdot \frac{\cos x}{1 + \cos x} = \dots$   
 $\Rightarrow \frac{1 - \cos x}{1 + \cos x} = \dots \checkmark$

9.  $1 + \cot^2 x = \csc^2 x$   
 $\Rightarrow \csc^2 x = \csc^2 x$   
 (id sheet)  
 $\Rightarrow \frac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$   
 $\Rightarrow \frac{1}{\csc^2 x} - \frac{1}{\csc^2 x} = \cos^2 x$   
 $\Rightarrow 1 - \sin^2 x = \cos^2 x$   
 $\Rightarrow \cos^2 x = \cos^2 x \checkmark$

10.  $\frac{\cot x - 1}{\cot x + 1} = \frac{1 - \tan x}{1 + \tan x}$   
 $\Rightarrow \frac{\frac{1}{\tan x} - 1}{\frac{1}{\tan x} + 1} = \dots$   
 $\Rightarrow \frac{1 - \tan x}{1 + \tan x} = \dots$   
 $\Rightarrow \frac{1 - \tan x}{1 + \tan x} = \frac{1 - \tan x}{1 + \tan x}$

2.  $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$   
 $\Rightarrow \sin x \tan x + \sin x \cot x + \cos x \tan x + \cos x \cot x = \dots$   
 $\Rightarrow \left(\frac{\sin^2 x}{\cos x} + \cos x\right) + \left(\sin x + \frac{\cos^2 x}{\sin x}\right) = \dots$   
 $\Rightarrow \frac{\sin^2 x + \cos^2 x}{\cos x} + \frac{\sin^2 x + \cos^2 x}{\sin x} = \dots$

$\Rightarrow \frac{1}{\cos x} + \frac{1}{\sin x} = \dots$   
 $\Rightarrow \sec x + \csc x = \sec x + \csc x \checkmark$

$$13. \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cos x.$$

$$\Rightarrow \frac{\sin^2 x \sin x + \cos^2 x \cos x}{\sin x + \cos x} = 1$$

$$\Rightarrow \frac{(1 - \cos^2 x) \sin x + (1 - \sin^2 x) \cos x}{\sin x + \cos x} = 1$$

$$\Rightarrow \frac{\sin x - \cos^2 x \sin x + \cos x - \cos x \sin^2 x}{\sin x + \cos x} = 1$$

$$\Rightarrow \frac{\sin x + \cos x - \cos^2 x \sin x - \cos x \sin^2 x}{\sin x + \cos x} = 1$$

$$\Rightarrow \frac{(\sin x + \cos x) - \cos x \sin x (\cos x + \sin x)}{\sin x + \cos x} = 1$$

$$\Rightarrow \frac{(\cancel{\sin x + \cos x}) (1 - \cos x \sin x)}{\cancel{\sin x + \cos x}} = 1$$

$$\Rightarrow 1 - \cos x \sin x = 1 - \sin x \cos x$$

$$\Rightarrow 1 - \sin x \cos x = 1 - \sin x \cos x \quad \checkmark$$

$$15. \frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = 4 \tan x \sec x.$$

$$\Rightarrow \frac{(1 + \sin x)(1 + \sin x) - (1 - \sin x)(1 - \sin x)}{(1 - \sin x)(1 + \sin x)} = 1$$

$$\Rightarrow \frac{1 + 2\sin x + \sin^2 x - (1 - 2\sin x + \sin^2 x)}{1 - \sin^2 x} = 1$$

$$\Rightarrow \frac{4\sin x}{\cos^2 x} = 1$$

$$\Rightarrow \frac{4\sin x}{\cos x \cos x} = 1$$

$$14. \frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$$

$$\Rightarrow \frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x}$$

$$\Rightarrow \frac{\cos x + 1}{\sin^2 x} = \frac{\csc x + \csc x \cos x}{1 - \cos^2 x}$$

$$\Rightarrow 1 = \frac{\csc x (1 + \cos x)}{\sin^2 x}$$

$$\Rightarrow 1 = \frac{1 + \cos x}{\sin x \sin^2 x}$$

$$\Rightarrow \frac{\cos x + 1}{\sin^3 x} = \frac{\cos x + 1}{\sin^3 x} \quad \checkmark$$

$$16. \csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$$

$$(\csc^2 x + \cot^2 x)(\csc^2 x - \cot^2 x) = 1$$

$$(\csc^2 x + \cot^2 x)(1) = 1$$

$$\csc^2 x + \cot^2 x = \csc^2 x + \cot^2 x \quad \checkmark$$

$$4 \tan x \sec x = 4 \tan x \sec x \quad \checkmark$$

$$17. \frac{\sin^2 x}{\cos^2 x + 3 \cos x + 2} = \frac{1 - \cos x}{2 + \cos x}$$

$$\Rightarrow \frac{\sin^2 x}{(\cos x + 1)(\cos x + 2)} = 1$$

$$\Rightarrow \frac{1 - \cos^2 x}{(\cos x + 1)(\cos x + 2)} = 1$$

$$\Rightarrow \frac{(1 + \cos x)(1 - \cos x)}{(\cos x + 1)(\cos x + 2)} = 1$$

$$\Rightarrow \frac{1 - \cos x}{2 + \cos x} = \frac{1 - \cos x}{2 + \cos x} \checkmark$$

$$18. \frac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$$

$$\Rightarrow \frac{\tan x + \tan y}{\frac{1}{\tan x} + \frac{1}{\tan y}} = 1$$

$$\Rightarrow \frac{\tan x + \tan y}{\tan y + \tan x} = 1$$

$$\Rightarrow \tan x + \tan y = \tan x + \tan y \checkmark$$

$$19. \frac{1 + \tan x}{1 - \tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$\Rightarrow \frac{1 + \tan x}{1 - \tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$\Rightarrow \frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$\Rightarrow \frac{\cos x + \sin x}{\cos x - \sin x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$20. (\sin x - \tan x)(\cos x - \cot x) = (\sin x - 1)(\cos x - 1)$$

$$\Rightarrow \sin x \cos x - \sin x \cot x - \tan x \cos x + \tan x \cot x = 1$$

$$\Rightarrow \sin x \cos x - \cos x - \sin x + 1 = 1$$

$$\Rightarrow \cos x (\sin x - 1) - \sin x + 1 = 1$$

$$\Rightarrow \cos x (\sin x - 1) - (1 - \sin x) = 1$$

$$\Rightarrow \cos x (\sin x - 1) - 1 (\sin x - 1) = 1$$

$$\Rightarrow (\cos x - 1)(\sin x - 1) = (\sin x - 1)(\cos x - 1)$$

$$\Rightarrow (\sin x - 1)(\cos x - 1) = (\sin x - 1)(\cos x - 1) \checkmark$$