

PreCalculus CH 6 Practice Test

Answer the following questions.

1. With regards to trigonometry, angles can be viewed as a circular rotation.

2. To convert from **radians to degrees**, we multiply by $\frac{180}{\pi}$. To convert from **degrees to radians**, we multiply by $\frac{\pi}{180}$.

3. Convert the following degree measures into radians.

a. 62°

$$\frac{62 \cdot \pi}{180} = \frac{31\pi}{90}$$

b. 30°

$$\frac{\pi}{6}$$

c. 1290°

$$\frac{1290 \cdot \pi}{180} = \frac{43\pi}{6}$$

d. -75°

$$-75 \cdot \frac{\pi}{180} = \frac{-15\pi}{36} = \frac{-5\pi}{12}$$

e. 7.5°

$$\frac{7.5\pi}{180} = \frac{15\pi}{360} = \frac{\pi}{24}$$

4. Convert the following radians into degrees.

a. $\frac{7\pi}{6}$

$$\frac{7\pi}{6} \cdot \frac{180}{\pi} = 210^\circ$$

b. $\frac{11\pi}{3}$

$$\frac{11\pi}{3} \cdot \frac{180}{\pi} = 660^\circ$$

c. -1.2

$$-1.2 \cdot \frac{180}{\pi} = -68.75^\circ$$

d. $-\frac{13\pi}{12}$

$$-\frac{13\pi}{12} \cdot \frac{180}{\pi} = -195^\circ$$

e. 3.4

$$3.4 \cdot \frac{180}{\pi} = 194.8^\circ$$

5. Find a positive and a negative **coterminal** angle for the given angle measure.

a. 50°

$$50 + 360 = 410^\circ$$

$$50 - 360 = -310^\circ$$

b. $\frac{3\pi}{4}$

$$\frac{3\pi}{4} \pm \frac{2\pi}{1} = \frac{7\pi}{4}$$

$$\frac{3\pi}{4} \pm \frac{2\pi}{1} = \frac{-5\pi}{4}$$

c. $-\frac{\pi}{4}$

$$-\frac{\pi}{4} \pm \frac{2\pi}{1} = \frac{7\pi}{4}$$

$$-\frac{\pi}{4} \pm \frac{2\pi}{1} = \frac{-9\pi}{4}$$

d. -45°

$$-45 \pm 360 = 315^\circ$$

$$-45 \pm 360 = -405^\circ$$

e. $\frac{11\pi}{6}$

$$\frac{11\pi}{6} \pm \frac{2\pi}{1} = \frac{23\pi}{6}$$

$$\frac{11\pi}{6} \pm \frac{2\pi}{1} = \frac{-\pi}{6}$$

6. Find an angle between 0° and 360° that is coterminal with the given angle.

a. 733°

$$733 - 2(360) = 13^\circ$$

b. 1110°

$$1110 - 3(360) = 30^\circ$$

c. -800°

$$-800 + 3(360) = 280^\circ$$

7. Find an angle between 0 and 2π that is coterminal with the given angle.

a. $\frac{5\pi}{3}$

$$\frac{5\pi}{3}$$

b. $-\frac{7\pi}{3}$

$$-\frac{7\pi}{3} + 2(2\pi) = \frac{5\pi}{3}$$

c. $\frac{51\pi}{2}$

$$\frac{51\pi}{2} - 12(2\pi) = \frac{3\pi}{2}$$

$$\frac{51}{2} = 25 \frac{1}{2} \quad (24)$$

8. Find the length of an arc that subtends a central angle of 45° in a circle of radius 10m.

$s = r\theta$ need to change to radians $\frac{45}{180} \cdot \pi = \frac{\pi}{4} = \theta$

$$s = 10 \left(\frac{\pi}{4} \right) = \frac{10\pi}{4} = \frac{5\pi}{2}$$

9. A central angle θ in a circle of radius 5m is subtended by an arc of length 6m. Find the measure of θ in degrees and in radians.

$s = r\theta$
 $\frac{6}{5} = \theta$
 rad = $\frac{6}{5}$
 deg = 678.6

$\frac{6}{5} \cdot \frac{180}{\pi} = \text{deg}$ ≈ 678.6

10. Find the radius of the circle if an arc of length 6m on the circle subtends a central angle of $\frac{\pi}{6}$ rad.

$s = r\theta$
 $6 = r \left(\frac{\pi}{6} \right)$
 $\Rightarrow r = \frac{36}{\pi} = 11.5$

11. Find the area of a sector with central angle 1 rad in a circle of radius 10m.

$A = \frac{1}{2} r^2 \theta \Rightarrow A = \frac{1}{2} (10^2) (1) = 50$

12. Find the area of a sector with central angle of 60° in a circle of radius 3m.

need to change to rad. first $\frac{60}{180} \cdot \pi = \frac{\pi}{3} = \theta$

$A = \frac{1}{2} r^2 \theta = \frac{1}{2} (3^2) \left(\frac{\pi}{3} \right) = 4.71$

13. Sketch a triangle that has acute angle θ , and find the other five trig ratios of θ .

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a. $\sin \theta = \frac{3}{5}$ opp/hyp

b. $\cos \theta = \frac{4}{5}$

c. $\csc \theta = \frac{5}{3}$

$\sin \theta = \frac{12}{13}$

$a^2 + b^2 = 5^2 \Rightarrow b^2 = 16 \Rightarrow b = 4$

$9^2 + b^2 = 40^2 \Rightarrow b^2 = 1519 \Rightarrow b \approx 39$

$a^2 + 12^2 = 13^2 \Rightarrow a^2 = 25 \Rightarrow a = 5$

Trig ratios for θ (3-4-5 triangle):
 $\sin \theta = \frac{3}{5}$, $\cos \theta = \frac{4}{5}$, $\tan \theta = \frac{3}{4}$, $\csc \theta = \frac{5}{3}$, $\sec \theta = \frac{5}{4}$, $\cot \theta = \frac{4}{3}$

Trig ratios for θ (9-39-40 triangle):
 $\sin \theta = \frac{39}{40}$, $\cos \theta = \frac{9}{40}$, $\tan \theta = \frac{39}{9}$, $\csc \theta = \frac{40}{39}$, $\sec \theta = \frac{40}{9}$, $\cot \theta = \frac{9}{39}$

Trig ratios for θ (5-12-13 triangle):
 $\sin \theta = \frac{12}{13}$, $\cos \theta = \frac{5}{13}$, $\tan \theta = \frac{12}{5}$, $\csc \theta = \frac{13}{12}$, $\sec \theta = \frac{13}{5}$, $\cot \theta = \frac{5}{12}$

14. Solve ΔABC , where $\angle A = 20^\circ$, $\angle C = 25^\circ$, and $c = 80.4$

$\frac{\sin C}{c} = \frac{\sin A}{a} \Rightarrow \frac{\sin 25}{80.4} = \frac{\sin 20}{a} \Rightarrow a = 65.1$

$\frac{\sin C}{c} = \frac{\sin B}{b} \Rightarrow \frac{\sin 25}{80.4} = \frac{\sin B}{b} \Rightarrow b = 134.5$

$\angle B = 180 - 20 - 25 = 135^\circ$

$a = 65.1, B = 135^\circ, b = 134.5$

15. Solve ΔABC if $\angle A = 45.3^\circ$, $a = 167.1$, and $b = 185.2$

① $\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \frac{\sin 45.3}{167.1} = \frac{\sin B}{185.2} \Rightarrow \sin B = 0.7878 \Rightarrow B = 52.0$

② $180 - 52 = 128 = \angle C$

$\frac{\sin 82.7}{c} = \frac{\sin 52}{185.2} \Rightarrow c = 233.1$

$\frac{\sin 6.7}{c} = \frac{\sin 128}{185.2} \Rightarrow c = 27.4$

16. Solve $\triangle ABC$ if $\angle A = 42^\circ$, $a = 70$, and $b = 122$

(122) $\frac{\sin 42}{70} = \frac{\sin B}{122}$ $\sin B = 1.17 > 1$
 impossible.

No Sol.

17. Solve $\triangle ABC$ if $\angle A = 45^\circ$, $a = 7\sqrt{2}$, and $b = 7$

(7) $\frac{\sin 45}{7\sqrt{2}} = \frac{\sin B}{7}$ $\frac{\sin 105}{c} = \frac{\sin 30}{7}$

$0.5 = \sin B$
 $B = 30^\circ$ $C = 105^\circ$

$c = 13.5$

(2) $180 - 30 = 150$

But, $150 + 45 > 180$

Impossible

18. Solve $\triangle ABC$ if $a = 5$, $b = 8$, and $c = 12$

$a^2 = b^2 + c^2 - 2bc \cos A$

$25 = 64 + 144 - 2(8)(12) \cos A$

$-183 = -192 \cos A$

$\cos A = 0.953125$

$A = 17.6^\circ$

$C = 133.4^\circ$

$b^2 = a^2 + c^2 - 2ac \cos B$

$64 = 25 + 144 - 2(5)(12) \cos B$

$-105 = -120 \cos B$

$0.875 = \cos B$

$B = 29^\circ$

19. Solve $\triangle ABC$ if $\angle A = 46.5^\circ$, $b = 10.5$, and $c = 18$

$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = (10.5)^2 + 18^2 - 2(10.5)(18) \cos 46.5$

$a^2 = 174.1$

$a = 13.2$

$b^2 = a^2 + c^2 - 2ac \cos B$

$110.25 = 174.24 + 324 - 2(13.2)(18) \cos B$

$-388 = -475.2 \cos B$

$\cos B = 0.82$

$B = 35.3^\circ$

$C = 98.2^\circ$

20. Find the area of the triangle whose sides have lengths: $a = 9$, $b = 12$, $c = 15$

$S = \frac{1}{2}(9+12+15)$

$= 18$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-9)(18-12)(18-15)}$

$= \sqrt{2916} = 54$

21. A ceiling fan with 16-in. blade rotates at 45rpm.

a. Find the angular speed of the fan in rad/min.

$\omega = \frac{\theta}{t}$

$45 \cdot 2\pi = 90\pi = \theta \text{ rad.}$

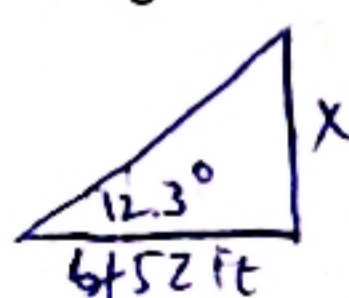
$\omega = \frac{90\pi}{1} = 90\pi \text{ rad/min.}$

b. Find the linear speed of the blades in in/min.

$v = \omega r$

$v = 90\pi(16) = 1440\pi = 4523.9 \text{ in/min.}$

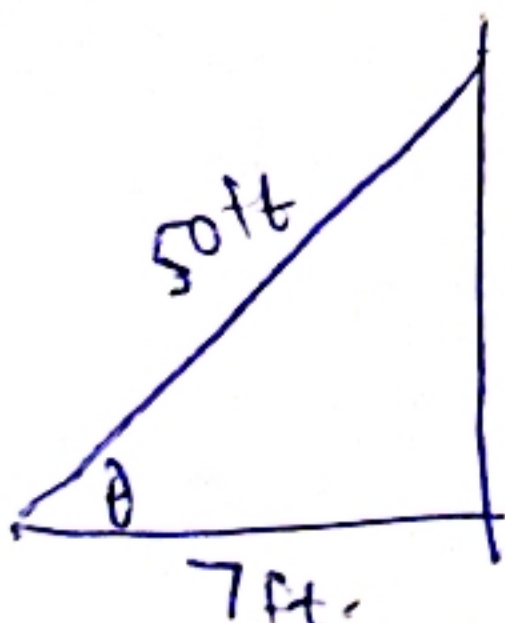
22. A giant redwood tree casts a shadow 452ft long. Find the height of the tree if the angle of elevation of the sun is 12.3° . (Hint: Draw the picture!)



$(452) \tan 12.3 = \frac{x}{452} (452)$

$x = 98.6 \text{ ft}$

23. A 50-ft ladder leans against a building. If the base of the ladder is 7ft from the base of the building, what is the angle formed by the ladder and the building? (Hint: Draw the picture!)



$\cos \theta = \frac{7}{50}$

$\cos^{-1}(\cos \theta) = 0.14$

$\theta \approx 82^\circ$