Name: Key **Period:** _____ **Date:** ___________

PreCalculus Chapter 2 Practice Test

Answer the following questions. No work is necessary unless it is specified.

1. Define function.

For every input there is exactly one output.

2. Let $f(x) = 2x^2 + 8x - 1$. Evaluate each function value.

a. $f(1)$ b. $f(3)$ c. $f(10)$ d. $f(a)$ e. $f(x^2)$

$f(1) = 2(1)^2+8(1)-1 = \boxed{9}$

$f(3) = 18+24-1 = \boxed{41}$

$f(10) = 200+80-1 = \boxed{279}$

$\boxed{f(a) = 2a^2+8a-1}$

$f(x^2) = 2(x^2)^2+8(x^2)-1 = \boxed{2x^4+8x^2-1}$

3. For the same function $f(x)$ from #2,

a. Find its domain.

D: $(-\infty, \infty)$

b. Complete the square and write it in the vertex form: $f(x) = a(x-h) + k$

$\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 2^2 = 4$

$f(x) = 2x^2+8x-1 \Rightarrow \frac{f(x)}{2} = x^2+4x-1 \Rightarrow \frac{f(x)}{2}+4 = (x^2+4x+4)-\frac{1}{2}$

$\Rightarrow \frac{f(x)}{2}+4 = (x+2)^2-\frac{1}{2} \Rightarrow \frac{f(x)}{2} = (x+2)^2 - \frac{9}{2} \Rightarrow \boxed{f(x) = 2(x+2)^2-9}$

c. Find its vertex and determine whether it's a maximum or a minimum point.

$\boxed{\text{Vertex} = (-2, -9)}$ minimum.

d. Describe the graph's change (shift, stretch, compress, etc.) from $f(x) = x^2$

Vertical stretch by S.F. 2, left 2 down 9.

4. Let $f(x) = x^2 - 4x - 5$

a. Find its domain.

$D: (-\infty, \infty)$.

b. Find the vertex and determine whether it is a maximum or a minimum point.

$x = \frac{-b}{2a} = \frac{4}{2(1)} = 2$ $y = f(2) = 2^2 - 4(2) - 5 = 4 - 8 - 5 = -9$

$\boxed{V = (2, -9)}$.

-5 1 -5 -4

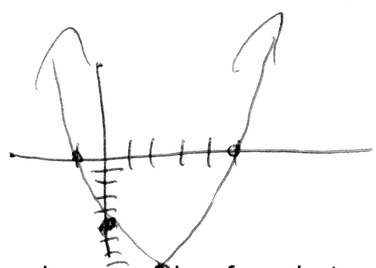
c. Find the x and the y intercepts.

y-int: $f(0) = 0 - 0 - 5 = \boxed{-5}$

$x+1=0$ $x-5=0$

X-int: $0 = x^2 - 4x - 5 = (x+1)(x-5) \Rightarrow \boxed{x = -1 \quad x = 5}$.

d. Use the vertex and the intercepts to sketch the graph the function.



5. Write the following equation for y in terms of x: $3x + 4y = 2$

$\frac{4y}{4} = \frac{2 - 3x}{4}$

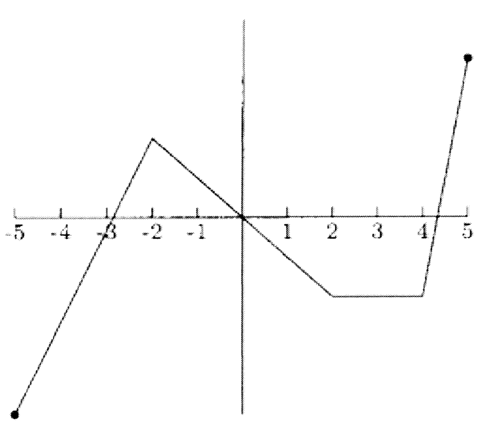
$\boxed{y = \frac{1}{2} - \frac{3}{4}x}$

6. Write the following equation for x in terms of y: $x - 2y - 3 = 0$

$+2y + 3$ $+2y + 3$.

$\boxed{x = 2y + 3}$

7. Use the graph to state the intervals in which the function is increasing, decreasing, and neither.



Inc: $[-5, -3]$, $[4, 5]$

Dec: $[-2, 2]$.

Neither: $[2, 4]$.

8. For the function $f(x) = 3x - 2$, determine the average rate of change between $x = 2, and\ x = 3$.

a b $f(2) = 3(2)-2 = 4$

$f(3) = 3(3)-2 = 7$

Avg. $= \frac{f(b)-f(a)}{b-a} = \frac{7-4}{3-2} = \frac{3}{1} = \boxed{3}$

9. Determine whether the following functions are one-to-one. If they are, find their inverse function.

a. $f(x) = -2x + 4$

$f(x_1) = f(x_2) \Rightarrow -2(x_1) \cancel{+4} = -2(x_2) \cancel{+4}$

$-4 \quad -4$

$\Rightarrow \frac{-2x_1}{-2} = \frac{-2x_2}{-2}$

$\Rightarrow x_1 = x_2$

One-to-one

$y = -2x + 4$

$x = -2y + 4$

$-\frac{x-4}{2} = y$

$\boxed{f^{-1}(x) = -\frac{x-4}{2}}$

b. $f(x) = \sqrt{x}$

$f(x_1) = f(x_2)$

$\Rightarrow \sqrt{x_1} = \sqrt{x_2}$

$x_1 = x_2$

one-to-one

$y = \sqrt{x}$

$x = \sqrt{y}$

$\boxed{y = x^2}$

$\boxed{f^{-1}(x) = x^2}$

c. $g(x) = x^2 - 2x$

~~f(x1) = f(x2) => x1^2 ... x2^2 ...~~

$g(2) = 4 - 4 = 0$

$g(0) = 0 - 0 = 0$

$2 \neq 0$

Not one-to-one.

d. $h(x) = x^3 + 8$

$h(x_1) = h(x_2) \Rightarrow x_1^3 + 8 = x_2^3 + 8$

$-8 \quad -8$

$\Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$

one-to-one

$y = x^3 + 8 \Rightarrow x = y^3 + 8 \Rightarrow x - 8 = y^3$

$\Rightarrow y = \sqrt[3]{x-8}$

$\Rightarrow \boxed{f^{-1}(x) = \sqrt[3]{x-8}}$

10. Let $f(x) = x - 3$ and $g(x) = 4x^2$. Find $f + g$, $f - g$, fg, $\frac{f}{g}$, $f \circ g$, $g \circ f$

$f + g = f(x) + g(x) = \boxed{x - 3 + 4x^2}$

$f - g = f(x) - g(x) = \boxed{x - 3 - 4x^2}$

$fg = f(x)g(x) = (x-3)(4x^2) = \boxed{4x^3 - 12x^2}$

$f/g = \frac{f(x)}{g(x)} = \boxed{\frac{x-3}{4x^2}}$

$f \circ g = f(g(x)) = f(4x^2) = \boxed{4x^2 - 3}$

$g \circ f = g(f(x)) = g(x-3) = 4(x-3)^2$

$= 4(x^2 - 6x + 9)$

$= \boxed{4x^2 - 24x + 36}$

11. Use $f(x) = 3x - 5$ and $g(x) = 2 - x^2$ to evaluate the following expressions.

a. $(f \circ g)(0)$

$f(g(0)) = f(2) = \boxed{1}$

b. $(f \circ g)(2)$

$f(g(2)) = f(-2) = \boxed{-11}$

$g \circ f = g(f(2))$

$= g(1) = \boxed{2}$

c. $(f \circ f)(3)$

$f(f(3)) = f(4)$

$= 3(4) - 5$

$= \boxed{7}$

d. $(g \circ f)(1)$

$g(f(1)) = g(-2)$

$= \boxed{-2}$

12. (T or F) Only one-to-one functions can have an inverse function.

13. (T or F) If a graph stretches vertically, then it also stretches horizontally.

14. (T or F) The set of all inputs (domain) of a function becomes the set of all outputs (range) for the inverse function.

15. (T or F) You can test for one-to-one-ness of a function using the vertical line test.

16. The effectiveness of a television commercial depends on how many times a viewer watches it. After some experiments an advertising agency found that if the effectiveness E is measured on a scale of 0 to 10, then $E(n) = \frac{2}{3}n - \frac{1}{90}n^2$, where *n* is the number of times a viewer watches a given commercial. For a commercial to have maximum effectiveness, how many times should a viewer watch it?

$E(n) = 2/3n - 1/90 n^2 \Rightarrow -\frac{1}{90}n^2 + 2/3 n$

max = vertex.

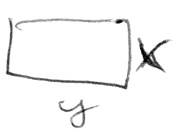
input: # of times watched. x

output: effectiveness. y

$x = \frac{-2/3}{2(-1/90)} = \frac{+2}{3} \cdot \frac{15}{1} = \boxed{30}$

17. A gardener has 240 feet of fencing to fence in a rectangular vegetable garden. Find the dimensions of the largest area she can fence. What is the maximum area?

P = 240



$2x + 2y = 240$

$\Rightarrow x + y = 120$

$y = 120 - x$

Area = length × width

$= xy.$

$A(x) = x(120 - x) = \boxed{120x - x^2}$

input: dimension x

output: Area y

$x = \frac{-b}{2a} = \frac{-120}{-2} = 60.$

$A(60) = 60(120 - 60) = \boxed{3600}$

18. A hockey team plays in an arena with a seating capacity of 10,500 spectators. With the ticket price set at $10, average attendance at recent games has been 9000. A market survey indicates that for each dollar the ticket price is lowered, the average attendance increases by 1000.

let x = ticket price.

a. What ticket price is so high that no one attends, and hence no revenue is generated?

Function : $R(X) = X[(10-x)1000+9000] = X(10000-1000x+9000) = X(19000-1000x)$

$\Rightarrow \quad R(x) = 19000x - 1000x^2$

No Revenue $\Rightarrow R(x)=0$. So, $0 = 19000x - 1000x^2 = x(19000-1000x)$

$\Rightarrow x=0$ or $19000-1000x=0$

$\frac{\cancel{-1000}x}{\cancel{-1000}} = \frac{-19\cancel{000}}{-1\cancel{000}}$

$\boxed{x=\$19}$

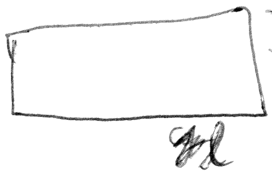
b. Find the price that maximizes revenue from ticket sales.

$R(x) = 19000x - 1000x$

input: ticket price x
output: Revenue y

$x = \frac{-19000}{-2(1000)} = \frac{19\cancel{000}}{2\cancel{000}} = \boxed{\$9.50}$

19. A rectangular building lot is three times as long as it is wide. Find a function that models its area A in terms of its width *w*.



$3w = l$

$\Rightarrow A(w) = w(3w) = \boxed{3w^2}.$

20. Find a function that models the radius *r* of a circle in terms of its area *A*.

$\text{Area} = \pi r^2 \Rightarrow r^2 = A/\pi \Rightarrow r = \sqrt{\frac{A}{\pi}},$

$\boxed{r(A) = \sqrt{A/\pi}}$

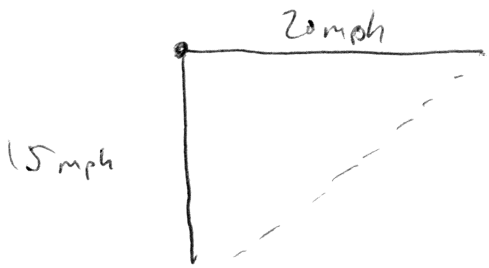
21. Find a function that models the area *A* of a circle in terms of its circumference *C*.

$C = 2\pi r \Rightarrow r = \frac{C}{2\pi}$

$A = \pi r^2.$

$\Rightarrow A(C) = \pi\left(\frac{C}{2\pi}\right)^2 = \pi\left(\frac{C^2}{4\pi^2}\right) = \boxed{\frac{C^2}{4\pi}}$

22. Two ships leave port at the same time. One sails south at 15mi/h and the other sails east at 20mi/h. Find a function that models the distance *D* between the ships in terms of the time *t* (in hours) elapsed since their departure.



$D^2 = a^2 + b^2$ (pythagorean).

$D = \sqrt{a^2+b^2}$

$a = 20t$

$b = 15t.$

$\Rightarrow D(t) = \sqrt{(20t)^2 + (15t)^2}$

$\Rightarrow D(t) = \sqrt{400t^2 + 225t^2}.$

$\Rightarrow D(t) = \sqrt{625t^2}$

$\Rightarrow \boxed{D(t) = 25t}$