Lesson 10-2

Ellipses

Objective

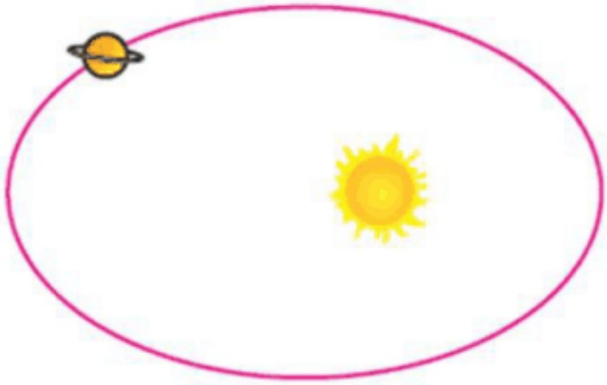
Students will…

- Be able to give a geometric definition of an ellipse.
- Be able to know the standard equation of ellipses.

Ellipse within a Cone

~~As seen from yesterday's video, a parabola can be cut out from a cone. Parabolas are easily found in the real world.~~

Ellipse

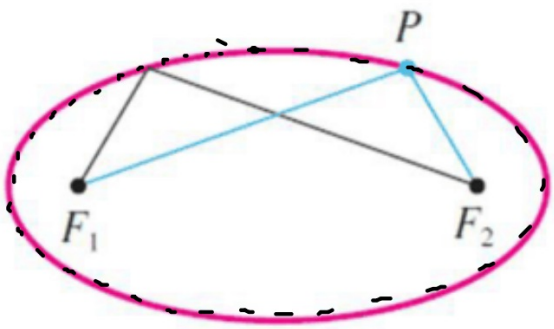


Ellipse

Here, we want to geometrically define what an ellipse is.

<u>Geometric Definition of an Ellipse</u>- An ellipse is the set of all points in the plane the sum whose distances from two fixed points F_1 and F_2 is a constant. These two fixed points are **foci** (plural of focus) of the ellipse.

Ex.



Equations and Graphs of Ellipses

Using the distance formula, we can see that parabolas have the following equations: ***for $a > b > 0$,***

Horizontal

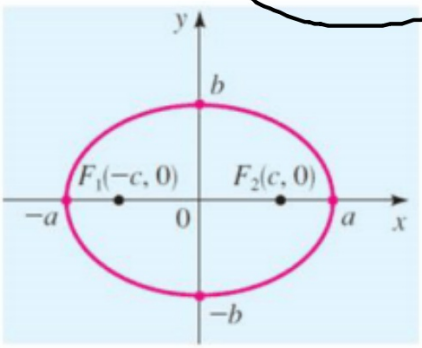
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Vertices: $(\pm a, 0)$; covertices = $(0, \pm b)$

Major Axis: Horizontal length 2a

Minor Axis: Vertical length $2b$

Foci: $(\pm c, 0)$, $c^2 = a^2 - b^2$



Vertical

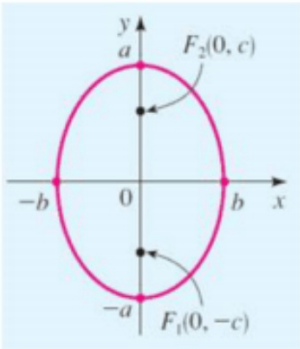
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$(0, \pm a)$; covertices = $(\pm b,$

Vertical length 2a

Horizontal length $2b$

$(0, \pm c)$, $c^2 = a^2 - b^2$



Example

An ellipse has the equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$ (horiz.).

Find the foci, vertices, and the lengths of the major and minor axes, and sketch the graph.

major: $2(3) = 6$

minor: $2(2) = 4$

Vertices: $(\pm a, 0)$

$= (\pm 3, 0)$

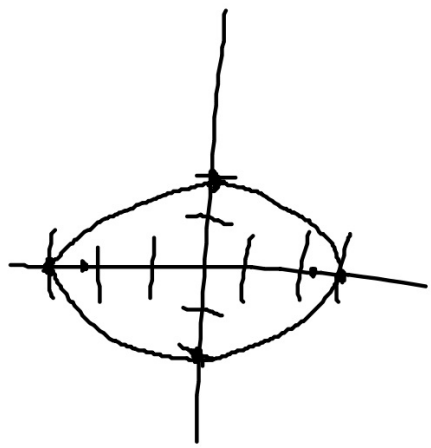
Covert: $(0, \pm b)$

$= (0, \pm 2)$

Foci: $(\pm c, 0) = (\pm\sqrt{5}, 0)$

$c^2 = a^2 - b^2 = 9 - 4 = 5$

$c = \pm\sqrt{5}$



Example

An ellipse has the equation $\frac{x^2}{36} + \frac{y^2}{25} = 1$ (horiz.).

Find the foci, vertices, and the lengths of the major and minor axes, and sketch the graph.

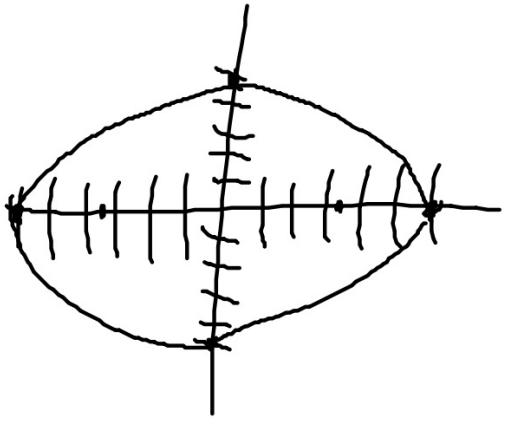
Vert: $(\pm 6, 0)$

Covert: $(0, \pm 5)$

Foci: $(\pm\sqrt{11}, 0)$

maj: 12

min: 10



Example

Find the foci of the ellipse $\frac{16x^2}{144} + \frac{9y^2}{144} = \frac{144}{144}$, and sketch its graph.

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{16} = 1 \quad \text{(vertical)}.$$

Foci: $(0, \pm c) = (0, \pm\sqrt{7})$

$c^2 = a^2 - b^2$

$c^2 = 16 - 9 = 7$

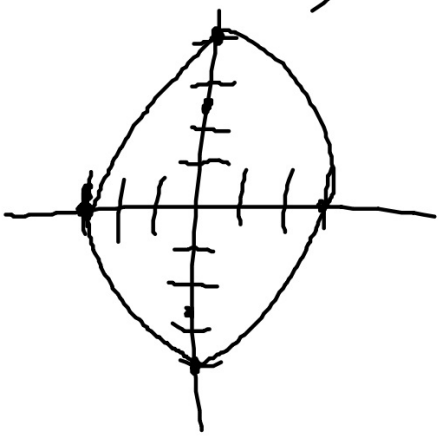
$c = \pm\sqrt{7}$

Vert: $(0, \pm a)$.

$= (0, \pm 4)$

Covert: $(\pm b, 0)$.

$= (\pm 3, 0)$.



Example

The vertices of an ellipse are $(\pm 4, 0)$ and foci are $(\pm 2, 0)$. Find its equation and sketch the graph.

$(\pm a, 0)$ $(\pm c, 0)$

horiz.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \boxed{\frac{x^2}{16} + \frac{y^2}{12} = 1}$$

$a^2 = 16$

$b^2 = 12$

$c^2 = 4$

$c^2 = a^2 - b^2$

$4 = 16 - b^2 \Rightarrow b^2 = 12$

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$$\frac{16x^2}{143} + \frac{9y^2}{143} = \frac{143}{143}$$

$$\Rightarrow \boxed{\frac{x^2}{\frac{143}{16}} + \frac{y^2}{\frac{143}{9}} = 1}$$

ex. $\frac{16x^2}{143} = \frac{16}{143} \cdot x^2 = x^2 \cdot \frac{16}{143}$

$= x^2 \div \frac{143}{16}$