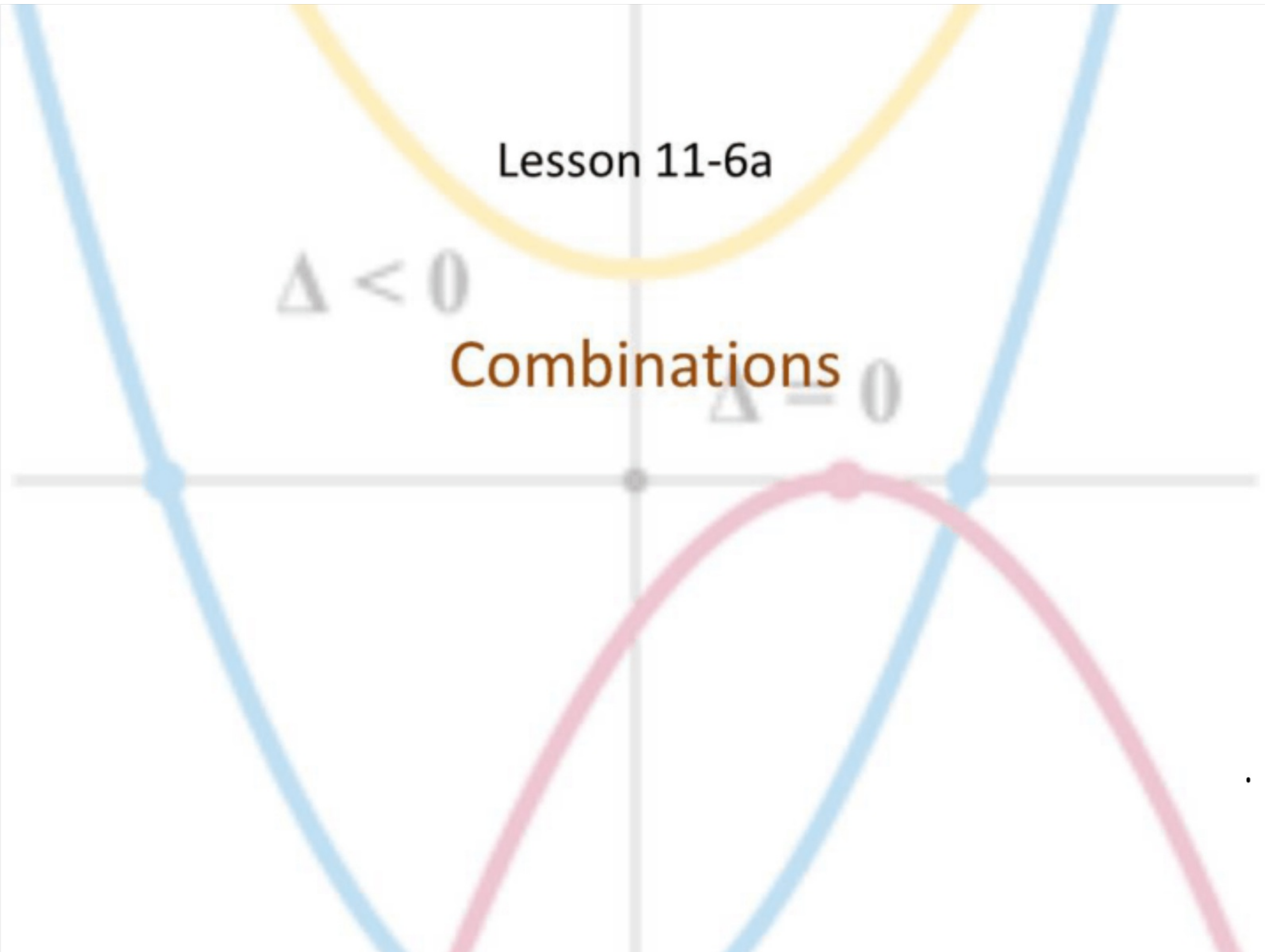
Lesson 11-6a

Combinations



Objective

Students will…

- Be able to calculate the combination of numbers.

Combinations

A **combination** of *r* elements of a set is any subset of *r* elements from the set (without regard to order). If the set has *n* elements, then the number of combinations of *r* elements is denoted by:

$$C(n,r) = {}_{n}C_{r} = \binom{n}{r}$$

We can read this as, "n choose r," denoting the number of ways to choose r elements of n elements. We can use any one of the three notations.

Example

For example, consider four elements, A, B, C, D. The combinations of these four elements taken three at a time, or in other words, the number of ways we can choose three of these four letters are…

ABC, ABD, ACD, BCD

Thus, $C(4,3) = {}_4C_3 = \binom{4}{3} = 4$

Now, this problem wasn't too hard to explicitly write all of the possible combinations out and then simply count them. However, what if instead of just those 4 letters, we were considering the entire alphabet? That would be too tedious!

Combinations

This is why it's useful to have a general formula for finding combinations. This formula is relatively simple to derive using the **counting principle**, which we won't be learning until the probability section. For now, consider the following:

The number of combinations of *n* objects taken *r* at a time is,

$$C(n,r) = {}_{n}C_{r} = \binom{n}{r} = \frac{n!}{r!(n-r)!}\text{, where}$$

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 1$$

Ex. $\binom{4}{3} = \frac{4!}{3!(4-3)!} = \frac{4!}{3!1!} = \frac{4\times3\times2\times1}{(3\times2\times1)(1)} = \frac{24}{6} = 4$

Example

Evaluate.

a. $\binom{9}{4} = \frac{9!}{4!(9-4)!} = \frac{9\times 8\times 7\times 6\times 5\times 4\times 3\times 2\times 1}{(4\times 3\times 2\times 1)(5\times 4\times 3\times 2\times 1)} = 3\times 7\times 6 = 126$

b. $\binom{100}{3} = \frac{100!}{3!(100-3)!} = \frac{100\times 99\times 98\times 97\times \cdots \times 1}{(3\times 2\times 1)(97\times \cdots \times 1)} = 100\times 33\times 49 = 161{,}700$

c. $\binom{90}{4} = \frac{90\times 89\times 88\times 87}{4\times 3\times 2\times 1} = 2{,}555{,}190$

Homework 4/23

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13–20 ■ Evaluate the expression.

13. $\binom{6}{4}$ **14.** $\binom{8}{3}$ **15.** $\binom{100}{98}$

$0! = 1$

16. $\binom{10}{5}$ **17.** $\binom{3}{1}\binom{4}{2}$ **18.** $\binom{5}{2}\binom{5}{3}$

19. $\binom{5}{0}+\binom{5}{1}+\binom{5}{2}+\binom{5}{3}+\binom{5}{4}+\binom{5}{5}$

20. $\binom{5}{0}-\binom{5}{1}+\binom{5}{2}-\binom{5}{3}+\binom{5}{4}-\binom{5}{5}$