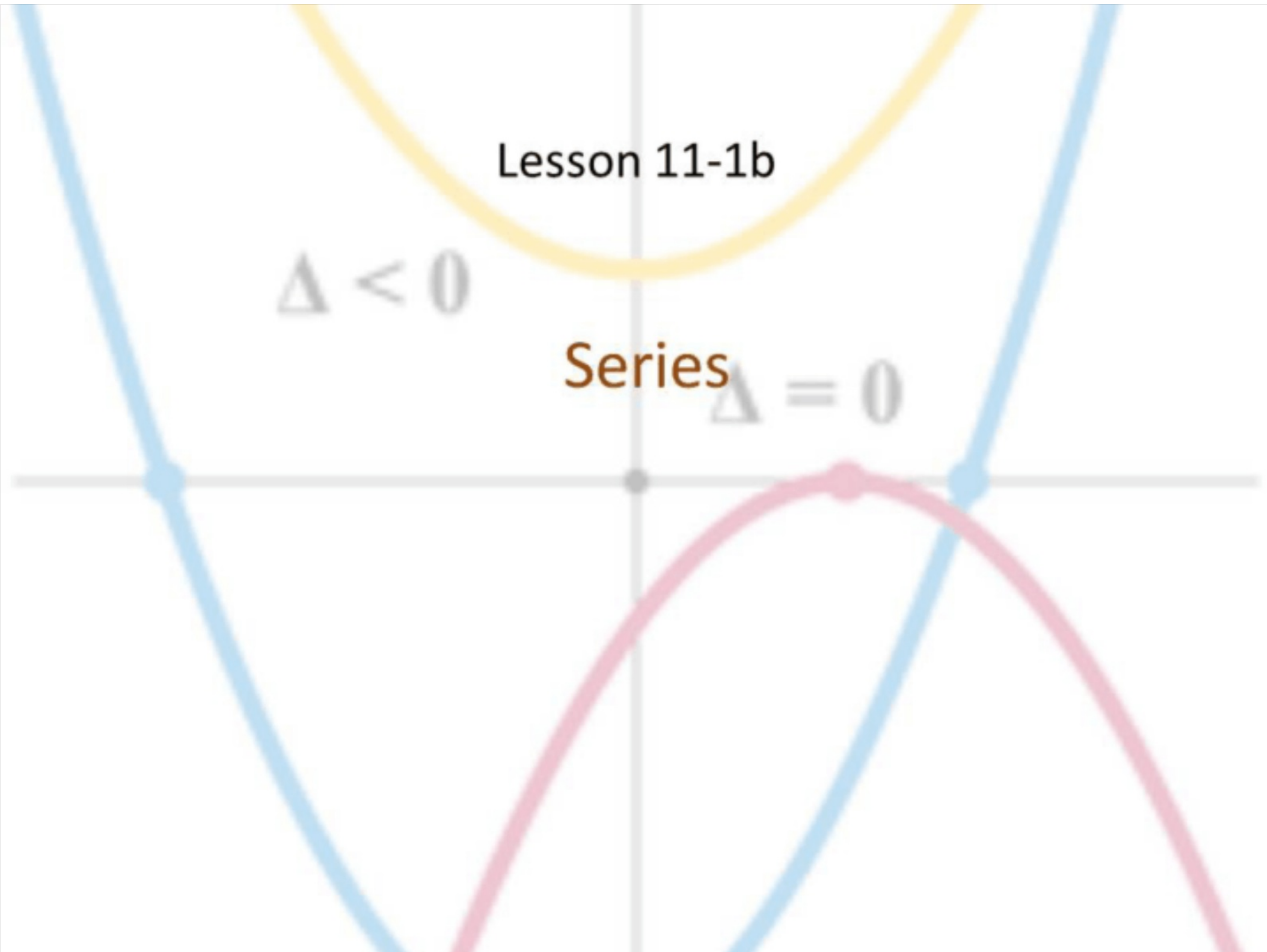
Lesson 11-1b

Series



Objective

Students will…

- Be able to define a series.
- Be able to calculate partial sums
- Be able to write series using summation notation.

Sequences

Many real-world processes generate list of numbers. For example, games scores, bank account numbers, etc. In mathematics, we call such lists **sequences**.

A sequence is a set of numbers written in a specific order ...

$$a_1, a_2, a_3, \dots, a_n$$

Here, a_1 is called the first term, a_2 is the second term, and so on. And a_n is the nth term of the sequence. Let's see how this look on a table:

1	2	3	4...	n
a_1	a_2	a_3	a_4 ...	a_n

Here, we can see that any given sequence can be written as a function.

Series

A **series** on the other hand, is the **<u>summation</u>** of a sequence. For example,

Sequence: $a_n = 1, 2, 3, 4, 5, 6$

Series: $\sum = 1 + 2 + 3 + 4 + 5 + 6 = 21$

As you can see, the above sequence has the last term, making it a **<u>finite</u>** sequence. So, naturally its series is also a **<u>finite series</u>**.

If a series is of an **<u>infinite</u>** sequence (no last term), then it is an **<u>infinite series</u>**.

Partial Sum

Finite series will always have a single sum of all the terms, because it has a last term. Infinite series on the other hand cannot have a single sum of all of the terms, because it simply never ends!

Therefore, for infinite series, we can only calculate its **partial sum**.

For the sequence: $a_1, a_2, a_3, a_4, \ldots$

the **partial sums** are:

$$S_1 = a_1$$
$$S_2 = a_1 + a_2 = S_1 + a_2$$
$$S_3 = a_1 + a_2 + a_3 = S_2 + a_3$$
$$\vdots$$
$$* \; S_n = a_1 + a_2 + a_3 + \cdots + a_n = S_{n-1} + a_n$$

S_1 is called the **first partial sum,** S_2 is called the **second partial sum**. So, S_n is called the **nth partial sum**. The sequence $S_1, S_2, S_3, \ldots, S_n, \ldots$ is called the **sequence of partial sums**.

Example

Find the first four partial sums and the nth partial sum of the sequence given by $a_n = \frac{1}{2^n}$ $= \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

$S_1 = \frac{1}{2}$

$S_2 = \frac{1}{2} + \frac{1}{4} = 3/4$

$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$

$S_4 = 7/8 + \frac{1}{16} = \frac{15}{16}$

$S_n = S_1, S_2, S_3, S_4, \ldots$

$S_n = \frac{1}{2}, 3/4, 7/8, 15/16, \ldots$

$S_n = \frac{2^n - 1}{2^n} = \frac{2^n}{2^n} - \frac{1}{2^n}$

$= \boxed{1 - \frac{1}{2^n}}$

Example

Find the first four partial sums and the nth partial sum of the sequence given by $a_n = 2n - 1$. $= 1, 3, 5, 7, \ldots$

$S_1 = 1$

$S_2 = 4$

$S_3 = 9$

$S_4 = 16$

$S_n = 1, 4, 9, 16, \ldots$

$\boxed{S_n = n^2}$

n	1	2	3	4
S_n	1	4	9	16

Summation (Sigma) Notation

We can represent series using the **summation notation** (summation since we are 'adding.'). For all summations we use the Greek alphabet, *sigma* ($\sum$).

Ex. Consider the following series, $a_1 + a_2 + a_3 + \cdots + a_n$. Using the summation notation, we have...

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \cdots + a_n$$

where $k = 1, 2, 3, \dots, n$. This is a case of a **finite** series. For an infinite series, say, $a_1 + a_2 + a_3 + \cdots$, we can write it as...

$$\sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + \cdots$$

Examples

Find each sum.

a. $\sum_{k=1}^{5} k^2 = 1+4+9+16+25 = \boxed{55}$

b. $\sum_{j=3}^{5} \frac{1}{j} = \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{20+15+12}{60} = \boxed{\frac{47}{60}}$

c. $\sum_{i=1}^{6} 2 = 2+2+2+2+2+2 = \boxed{12}$

Examples

Write each sum using sigma notation.

a. $\overset{0}{1^3} + \overset{1}{2^3} + \overset{2}{3^3} + \overset{3}{4^3} + \overset{4}{5^3} + \overset{5}{6^3} + \overset{6}{7^3}$

$$\boxed{\sum_{k=1}^{7} k^3} \text{ or } \sum_{k=2}^{8} (k-1)^3 \text{ or } \sum_{k=0}^{6} (k+1)^3$$

b. $\overset{3}{\sqrt{3}} + \overset{4}{\sqrt{4}} + \overset{5}{\sqrt{5}} + \overset{\cdots}{\cdots} + \overset{77}{\sqrt{77}}$

$$\boxed{\sum_{j=3}^{77} \sqrt{j}} \text{ or } \boxed{\sum_{j=1}^{75} \sqrt{j+2}}$$

Properties of Sums

Properties of Sums

Let $a_1, a_2, a_3, a_4, \ldots$ and $b_1, b_2, b_3, b_4, \ldots$ be sequences. Then for every positive integer n and any real number c, the following properties hold.

1. $\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$

2. $\sum_{k=1}^{n} (a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$

3. $\sum_{k=1}^{n} c a_k = c\left(\sum_{k=1}^{n} a_k \right)$

ex. $3(2)+4(2)$
$=2(3+4)$

Homework 4/17, 20

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