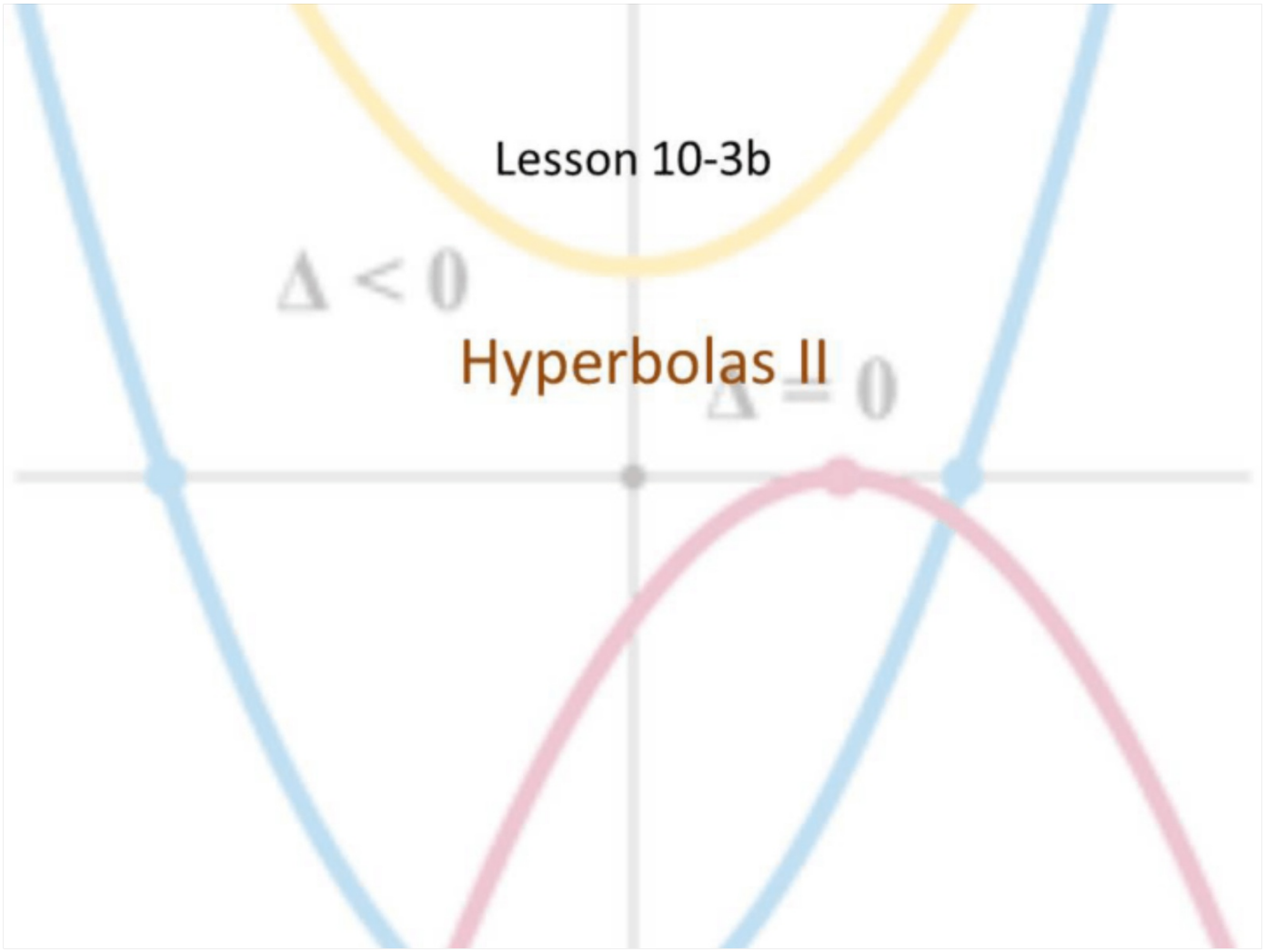

Lesson 10-3b
$\Delta < 0$
Hyperbolas II
$\Delta = 0$

Objective

Students will…

- Be able to derive the equation of standard hyperbolas, given the vertices and foci, and/or the asymptotes.

Equations and Graphs of Hyperbolas

Using the distance formula, we can see that parabolas have the following equations: ***for*** $a > 0$ and $b > 0$ (**not** $a > b$)

Horizontal

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

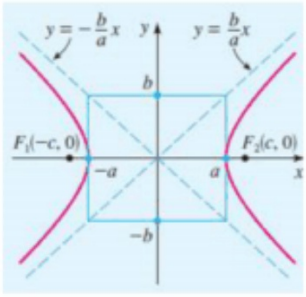
Vertices: $(\pm a, 0)$

Covertices: $(0, \pm b)$

Transverse Axis: Horizontal length 2a

Asymptotes: $y = \pm \frac{b}{a} x$

Foci: $(\pm c, 0)$, $c^2 = a^2 + b^2$



Vertical

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

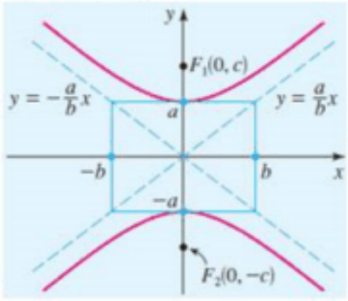
$(0, \pm a)$

$(\pm b, 0)$

Vertical length $2a$

$y = \pm \frac{a}{b} x$

$(0, \pm c)$, $c^2 = a^2 + b^2$

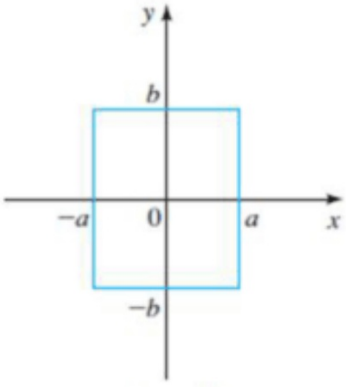


Sketching the Hyperbola

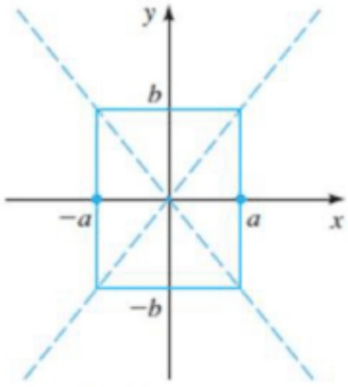
Here is a guidelines you can use to graph the hyperbola.

1. Sketch the **central box**, using the vertices and the covertices.
2. Sketch the **asymptotes**. These are the diagonals of the central box.
3. Plot the foci
4. Sketch the hyperbola.

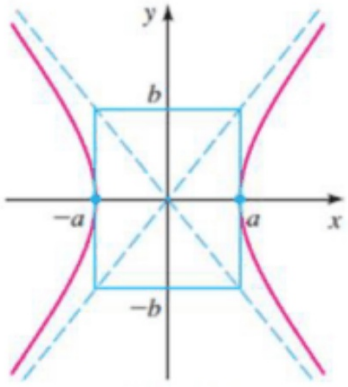
Ex.



(a) Central box



(b) Asymptotes



(c) Hyperbola

Example

Find the equation of the hyperbola with vertices $(\pm 3, 0)$ and foci $(\pm 4, 0)$. Sketch its graph.

$(\pm a, 0) \Rightarrow$ covert: $(0, \pm b)$

$= (0, \pm \sqrt{7})$

$\pm 2, 0$

horiz. $\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \boxed{\frac{x^2}{9} - \frac{y^2}{7} = 1}$

$a^2 = 3^2 = 9$

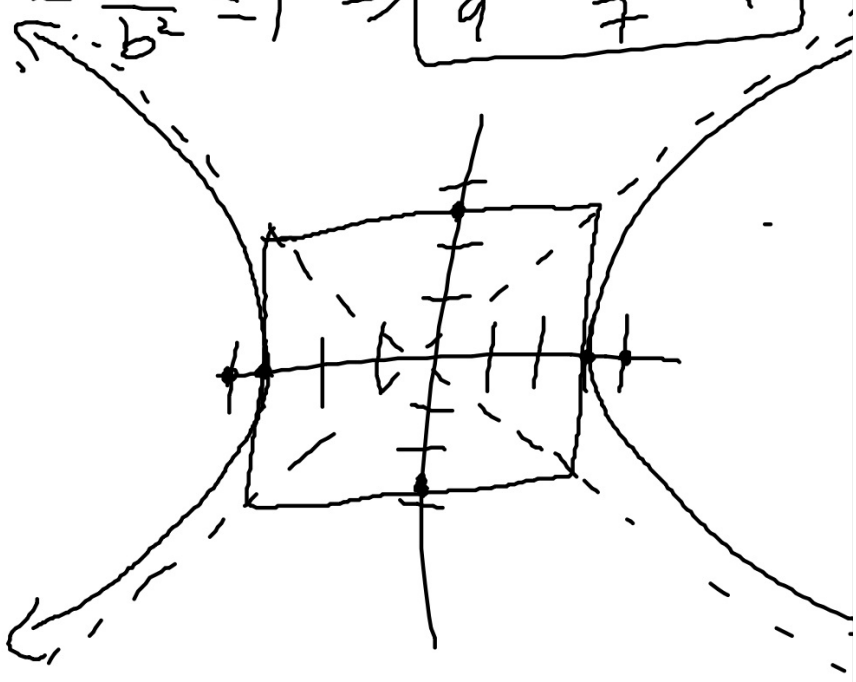
$b^2 = 7$

$c^2 = 4^2 = 16$

$c^2 = a^2 + b^2$

$16 = 9 + b^2$

$7 = b^2$



Example

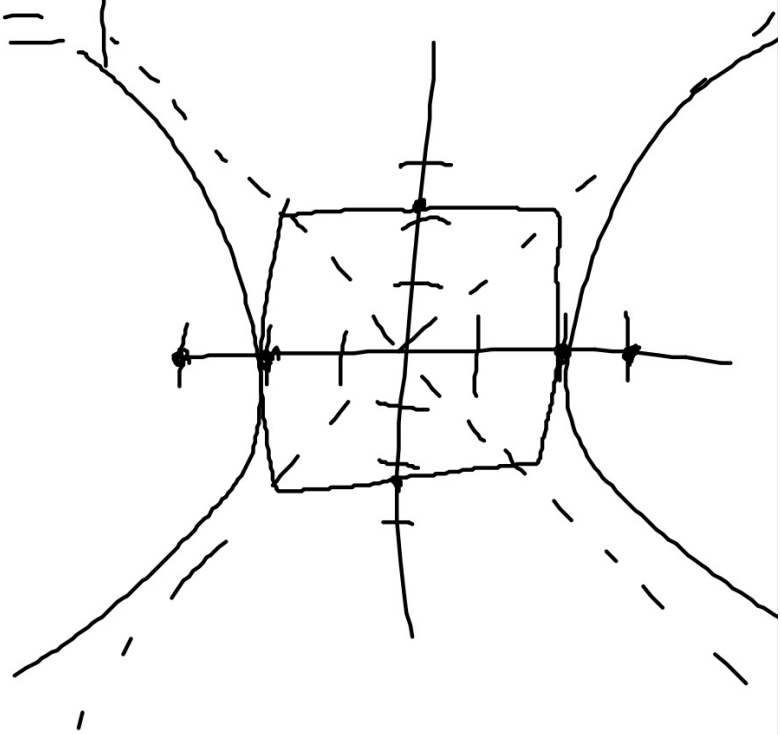
Find the equation of the hyperbola with vertices $(\pm 2, 0)$ and foci $(\pm 3, 0)$. Sketch its graph.

(over: $(0, \pm\sqrt{5})$)

horiz.

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

Foci: $(\pm 3, 0)$



Example

Find the equation and the foci of the hyperbola with vertices $(0, \pm 2)$ and asymptotes $y = \pm 2x$. Sketch the graph.

$(0, \pm a) \Rightarrow$ Vert $(\pm$

$y = \pm \frac{a}{b} x$

$y = \pm 2x \Rightarrow \frac{a}{b} = \frac{2}{1}$

$\Rightarrow \frac{2}{b} = \frac{2}{1} \Rightarrow$

Vertical $\quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

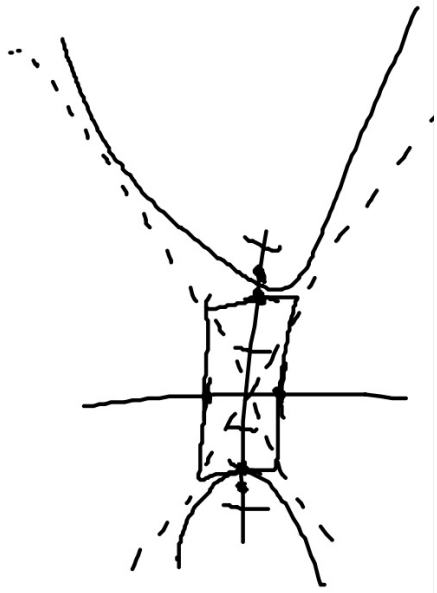
$a^2 = 4 \Rightarrow \frac{y^2}{4} - \frac{x^2}{1} = 1$

$b^2 = 1$

$c^2 = 5 \Rightarrow c = \pm\sqrt{5}$

$c^2 = a^2 + b^2$ $\quad$ Foci: $(0, \pm\sqrt{5})$

$c^2 = 4 + 1 = 5$



Example

Find the equation and the foci of the hyperbola with vertices $(0, \pm 4)$ and asymptotes $y = \pm 4x$. Sketch the graph.

Example

Find the equation of the hyperbola with vertices $(0, \pm 6)$, given that it passes through the point $(5, -9)$

$(0, \pm a)$ $\quad (x, y)$

Vertical

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$a^2 = 36$$

$$b^2 = 20$$

$$\Rightarrow \frac{y^2}{36} - \frac{x^2}{20} = 1$$

$$\Rightarrow \frac{(-9)^2}{36} - \frac{(5)^2}{b^2} = 1$$

$$\frac{81}{36} - \frac{25}{b^2} = \frac{36}{36} - \frac{81}{36}$$

$$+\frac{25}{b^2} \times \quad +\frac{45}{36}$$

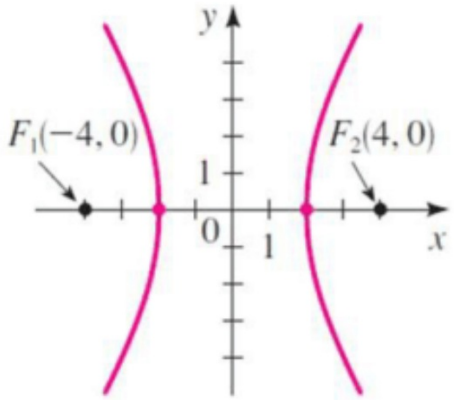
$$\Rightarrow 45b^2 = 900$$

$$b^2 = 20$$

Homework Problem

Find the equation for the hyperbola whose graph is shown.

17.



Homework Problem

Find the equation for the hyperbola with the given conditions.

38. Foci $(0, \pm 1)$, length of the transverse axis 1.

Homework 5/29

TB pgs. 768-769 #17, 19, 21, 27, 29, 31, 32, 34, 37, 38