3.1.2 Answers

1. $f(x) = 4 - x - 3x^2$
   Degree 2
   Leading term $-3x^2$
   Leading coefficient -3
   Constant term 4
   As $x \to -\infty,\ f(x) \to -\infty$
   As $x \to \infty,\ f(x) \to -\infty$

2. $g(x) = 3x^5 - 2x^2 + x + 1$
   Degree 5
   Leading term $3x^5$
   Leading coefficient 3
   Constant term 1
   As $x \to -\infty,\ g(x) \to -\infty$
   As $x \to \infty,\ g(x) \to \infty$

3. $q(r) = 1 - 16r^4$
   Degree 4
   Leading term $-16r^4$
   Leading coefficient -16
   Constant term 1
   As $r \to -\infty,\ q(r) \to -\infty$
   As $r \to \infty,\ q(r) \to -\infty$

4. $Z(b) = 42b - b^3$
   Degree 3
   Leading term $-b^3$
   Leading coefficient -1
   Constant term 0
   As $b \to -\infty,\ Z(b) \to \infty$
   As $b \to \infty,\ Z(b) \to -\infty$

5. $f(x) = \sqrt{3}x^{17} + 22.5x^{10} - \pi x^7 + \frac{1}{3}$
   Degree 17
   Leading term $\sqrt{3}x^{17}$
   Leading coefficient $\sqrt{3}$
   Constant term $\frac{1}{3}$
   As $x \to -\infty,\ f(x) \to -\infty$
   As $x \to \infty,\ f(x) \to \infty$

6. $s(t) = -4.9t^2 + v_0t + s_0$
   Degree 2
   Leading term $-4.9t^2$
   Leading coefficient -4.9
   Constant term s_0
   As $t \to -\infty,\ s(t) \to -\infty$
   As $t \to \infty,\ s(t) \to -\infty$

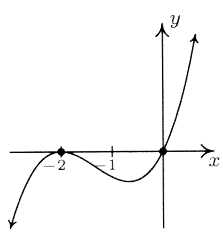
7. $P(x) = (x-1)(x-2)(x-3)(x-4)$
   Degree 4
   Leading term x^4
   Leading coefficient 1
   Constant term 24
   As $x \to -\infty,\ P(x) \to \infty$
   As $x \to \infty,\ P(x) \to \infty$

8. $p(t) = -t^2(3-5t)(t^2+t+4)$
   Degree 5
   Leading term $5t^5$
   Leading coefficient 5
   Constant term 0
   As $t \to -\infty,\ p(t) \to -\infty$
   As $t \to \infty,\ p(t) \to \infty$

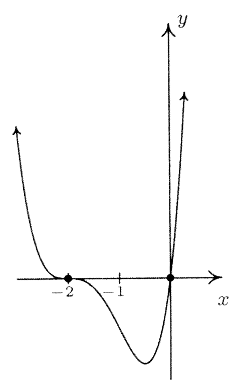
9. $f(x) = -2x^3(x+1)(x+2)^2$
Degree 6
Leading term $-2x^6$
Leading coefficient -2
Constant term 0
As $x \to -\infty,\ f(x) \to -\infty$
As $x \to \infty,\ f(x) \to -\infty$

10. $G(t) = 4(t-2)^2\left(t+\frac{1}{2}\right)$
Degree 3
Leading term $4t^3$
Leading coefficient 4
Constant term 8
As $t \to -\infty,\ G(t) \to -\infty$
As $t \to \infty,\ G(t) \to \infty$

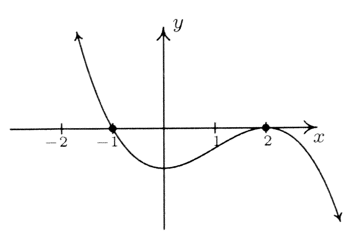
11. $a(x) = x(x+2)^2$
$x = 0$ multiplicity 1
$x = -2$ multiplicity 2



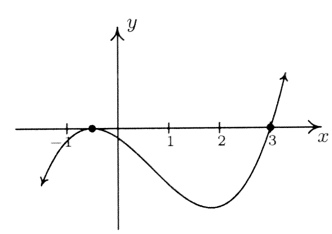
12. $g(x) = x(x+2)^3$
$x = 0$ multiplicity 1
$x = -2$ multiplicity 3



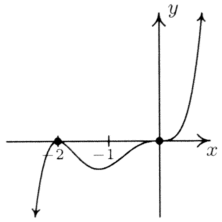
13. $f(x) = -2(x-2)^2(x+1)$
$x = 2$ multiplicity 2
$x = -1$ multiplicity 1



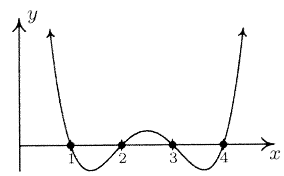
14. $g(x) = (2x+1)^2(x-3)$
$x = -\frac{1}{2}$ multiplicity 2
$x = 3$ multiplicity 1



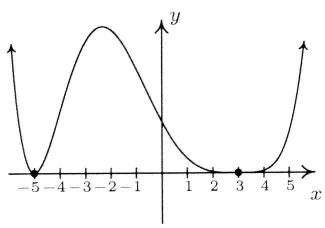
15. $F(x) = x^3(x+2)^2$
$x = 0$ multiplicity 3
$x = -2$ multiplicity 2



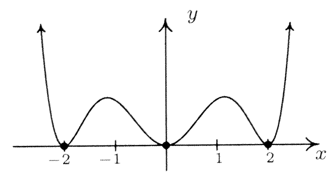
16. $P(x) = (x-1)(x-2)(x-3)(x-4)$
$x = 1$ multiplicity 1
$x = 2$ multiplicity 1
$x = 3$ multiplicity 1
$x = 4$ multiplicity 1



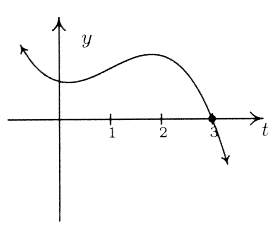
17. $Q(x) = (x+5)^2(x-3)^4$
$x = -5$ multiplicity 2
$x = 3$ multiplicity 4



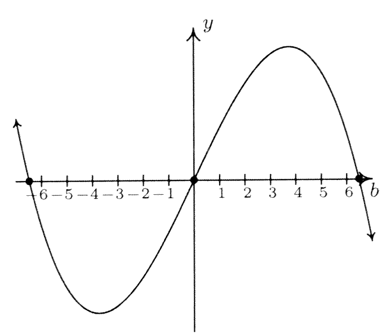
18. $f(x) = x^2(x-2)^2(x+2)^2$
$x = -2$ multiplicity 2
$x = 0$ multiplicity 2
$x = 2$ multiplicity 2



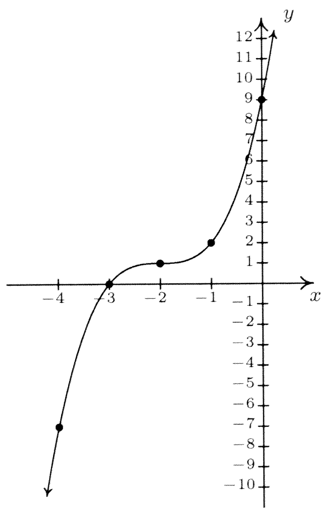
19. $H(t) = (3-t)\left(t^2+1\right)$
$x = 3$ multiplicity 1



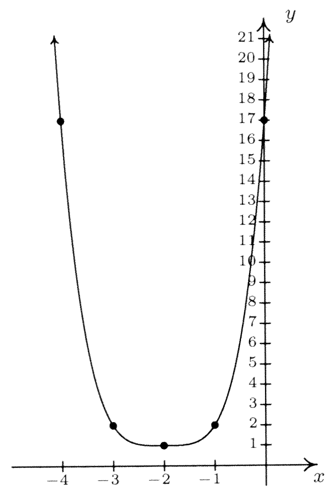
20. $Z(b) = b(42-b^2)$
$b = -\sqrt{42}$ multiplicity 1
$b = 0$ multiplicity 1
$b = \sqrt{42}$ multiplicity 1



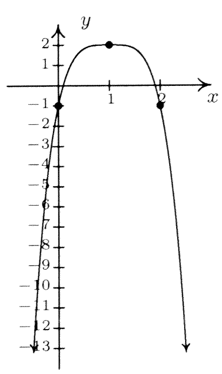
21. $g(x) = (x+2)^3 + 1$
domain: $(-\infty, \infty)$
range: $(-\infty, \infty)$



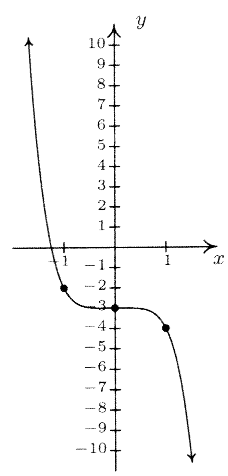
22. $g(x) = (x+2)^4 + 1$
domain: $(-\infty, \infty)$
range: $[1, \infty)$



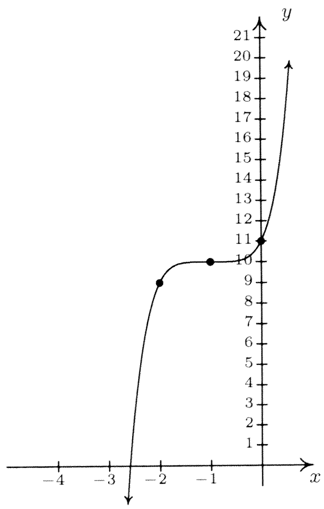
23. $g(x) = 2 - 3(x-1)^4$
domain: $(-\infty, \infty)$
range: $(-\infty, 2]$



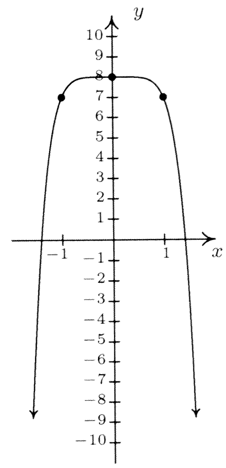
24. $g(x) = -x^5 - 3$
domain: $(-\infty, \infty)$
range: $(-\infty, \infty)$



25. $g(x) = (x+1)^5 + 10$
domain: $(-\infty, \infty)$
range: $(-\infty, \infty)$



26. $g(x) = 8 - x^6$
domain: $(-\infty, \infty)$
range: $(-\infty, 8]$



27. We have $f(-4) = -23$, $f(-3) = 5$, $f(0) = 5$, $f(1) = -3$, $f(2) = -5$ and $f(3) = 5$ so the Intermediate Value Theorem tells us that $f(x) = x^3 - 9x + 5$ has real zeros in the intervals $[-4, -3]$, $[0, 1]$ and $[2, 3]$.

28. $V(x) = x(8.5 - 2x)(11 - 2x) = 4x^3 - 39x^2 + 93.5x$, $0 < x < 4.25$. Volume is maximized when $x \approx 1.58$, so the dimensions of the box with maximum volume are: height ≈ 1.58 inches, width ≈ 5.34 inches, and depth ≈ 7.84 inches. The maximum volume is ≈ 66.15 cubic inches.

29. The calculator gives the location of the absolute maximum (rounded to three decimal places) as $x \approx 6.305$ and $y \approx 1115.417$. Since x represents the number of TVs sold in hundreds, $x = 6.305$ corresponds to 630.5 TVs. Since we can't sell half of a TV, we compare $R(6.30) \approx 1115.415$ and $R(6.31) \approx 1115.416$, so selling 631 TVs results in a (slightly) higher revenue. Since y represents the revenue in *thousands* of dollars, the maximum revenue is \$1,115,416.

30. $P(x) = R(x) - C(x) = -5x^3 + 35x^2 - 45x - 25$, $0 \leq x \leq 10.07$.

31. The calculator gives the location of the absolute maximum (rounded to three decimal places) as $x \approx 3.897$ and $y \approx 35.255$. Since x represents the number of TVs sold in hundreds, $x = 3.897$ corresponds to 389.7 TVs. Since we can't sell 0.7 of a TV, we compare $P(3.89) \approx 35.254$ and $P(3.90) \approx 35.255$, so selling 390 TVs results in a (slightly) higher revenue. Since y represents the revenue in *thousands* of dollars, the maximum revenue is \$35,255.

32. Making and selling 71 PortaBoys yields a maximized profit of \$5910.67.