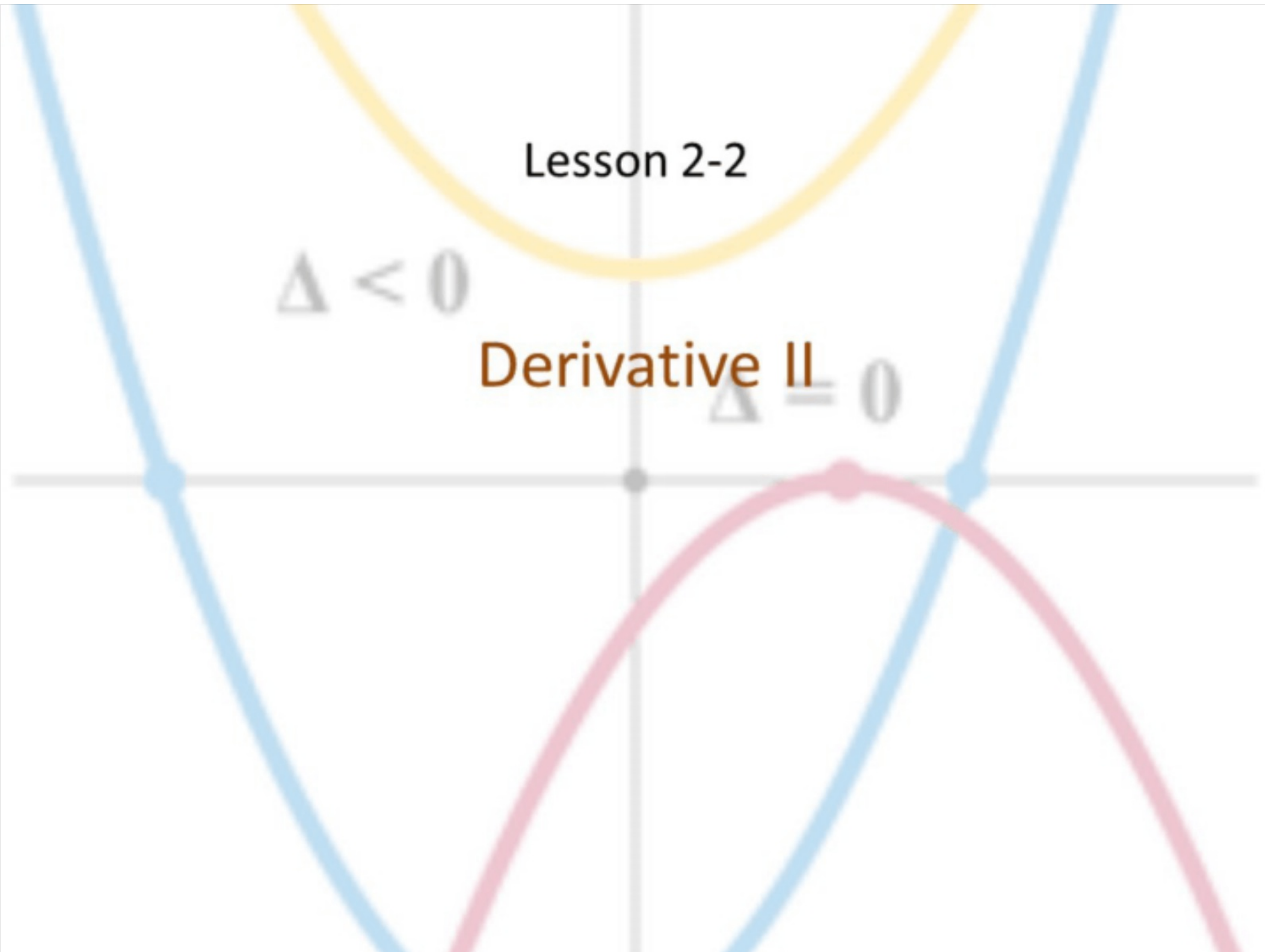
Lesson 2-2

Derivative II



Objective

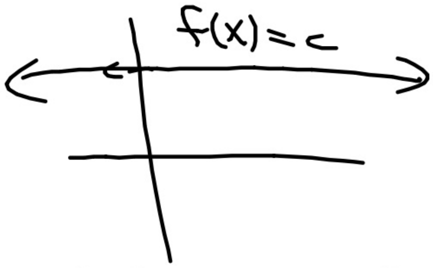
Students will…

- Be able to know and use the basic differentiation rule.
- Be able to use the derivatives to find the rates of change.
- Be able to relate derivative function to the velocity function.

The Constant Rule

You now know what differentiation is, and you can find derivative functions. As always, in mathematics, it is always a good thing if certain things can be generalized into an easier or simpler form. The first and foremost, the most basic result of differentiation is none other than the **constant rule**.

The derivative of a constant function is 0, that is, if c is a real number, then $\frac{d}{dx}[c] = 0$.



Think about the graph of any constant function. The slope is **always** zero.

The Power Rule

The most important and useful rule in derivative would be the **power rule**.

The Power Rule- If n is a rational number, then the function $f(x) = x^n$ is differentiable and $\frac{d}{dx}[x^n] = nx^{n-1}$

For f to be differentiable at $x = 0$, n must be a number such that x^{n-1} is defined on an interval containing 0.

$f(x) = x^{-2}$

$f'(x) = -2x^{-3}$

$f(x) = x^{-1}$

$f'(x) = -x^{-2}$

Proof of the Power Rule

$f(x) = x^n$

$$f'(x) = \lim_{h \to 0} \left(\frac{(x+h)^n - x^n}{h} = \frac{\cancel{x^n h^0} + c x^{n-1} h^1 + d x^{n-2} h^2 + \cdots + x^0 h^n \cancel{- x^n}}{h} \right)$$

$$\Rightarrow \lim_{h \to 0} \left(\frac{\cancel{h} (x^{n-1} \cancel{+ d x^{n-2} h + \cdots + x^0 h^{n-1}})}{\cancel{h}} \right)$$

$\vdots$

Examples

Find the derivative of the following:

a. $f(x) = x^3$

$\boxed{f'(x) = 3x^2}$

b. $g(x) = \sqrt[3]{x} = x^{1/3}$

$\boxed{g'(x) = \frac{1}{3} x^{-2/3}}$

Example

Find the derivative of $y = \frac{1}{x^2} = x^{-2}$

$y' = -2x^{-3}$

$y - y_1 = m(x - x_1)$

Deriv.

Example

Find the equation of a tangent line to the graph of $f(x) = x^2$ when $x = -2$.

(x_1, y_1) $(-2, 4)$

$f(-2) = 4$

$f'(x) = 2x$

$f'(-2) = -4 = m$

$$\boxed{y - 4 = -4(x+2)}$$

$$y = -4(x+2) + 4$$

Laws of Derivatives

1. $\frac{d}{dx}[c(fx)] = cf'(x)$, where c is a real number.

ex. $y = 2x^3 \Rightarrow y' = 6x^2$

$\Rightarrow y = 2x^3 \Rightarrow 2(y) = 2(x^3) \Rightarrow 2(y') = 2(3x^2) = 6x^2$

2. $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)] = f'(x) \pm g'(x)$

$f(x) = 3x^2 + 6x - 1$

$f'(x) = 6x + 6 - 0$

$g(x) = e^x + \sin(6\theta) + 3x^2$

***Note:** $\frac{d}{dx}[f(x) \times \div g(x)] \neq f'(x) \times \div g'(x)$

ex. $f(x) = x^2$, $g(x) = x^3$

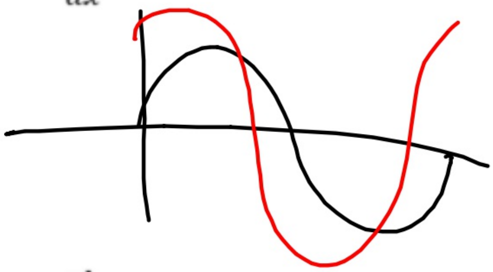
$(f \cdot g) = x^5 \Rightarrow (f \cdot g)' = 5x^4$

$f'(x) = 2x$ $g'(x) = 3x^2$

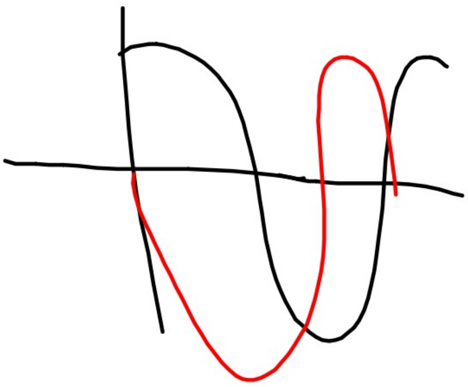
$(f' \cdot g') = 6x^3$

Trig Derivatives

1. $\frac{d}{dx}[\sin x] = \cos x$



2. $\frac{d}{dx}[\cos x] = -\sin x$



Trig Derivatives

3. $\frac{d}{dx}[\tan x] = \sec^2 x$

$$\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \text{T.B.C.}$$

4. $\frac{d}{dx}[\cot x] = -\csc^2 x$

$$\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \text{T.B.C.}$$

Velocity and Derivatives

Ex. If a billiard ball is dropped from a height of 100 feet, its height s at time t is given by the position function $s = -16t^2 + 100$, where s is measured in feet and t is measured in seconds. Find the average velocity (rate of change) over each of the following time intervals.

a. $[1,2]$ $\quad \frac{36-84}{2-1} = \frac{-48}{1} = -48$ ft/sec.

$x_1 \; x_2$

$y_1 = -16(1)^2 + 100$

$= 84$

$y_2 = -16(2)^2 + 100$

$= 36$

b. $[1, 1.5]$

$x_1 \; x_2$

Velocity and Derivatives

(instantaneous).

In context of motion, derivatives equate to the **velocity** of motion.

Ex. At time $t = 0$, a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$, where s is measured in feet and t is measured in seconds.

a. When does the diver hit the water?

$0 = -16t^2 + 16t + 32$

$0 = \cancel{-16}(t^2 - t - 2)$

$0 = \cancel{(t+1)}(t-2) \Rightarrow t = 2 \text{ sec.}$

b. What was the diver's velocity at impact?

$s'(t) = -32t + 16$

$s'(2) = -64 + 16 = -48 \text{ ft/sec}$

Homework Due 9/10

TB 2.2- #3-23 (odd), 39-51 (odd), 93, 94, 103, 104