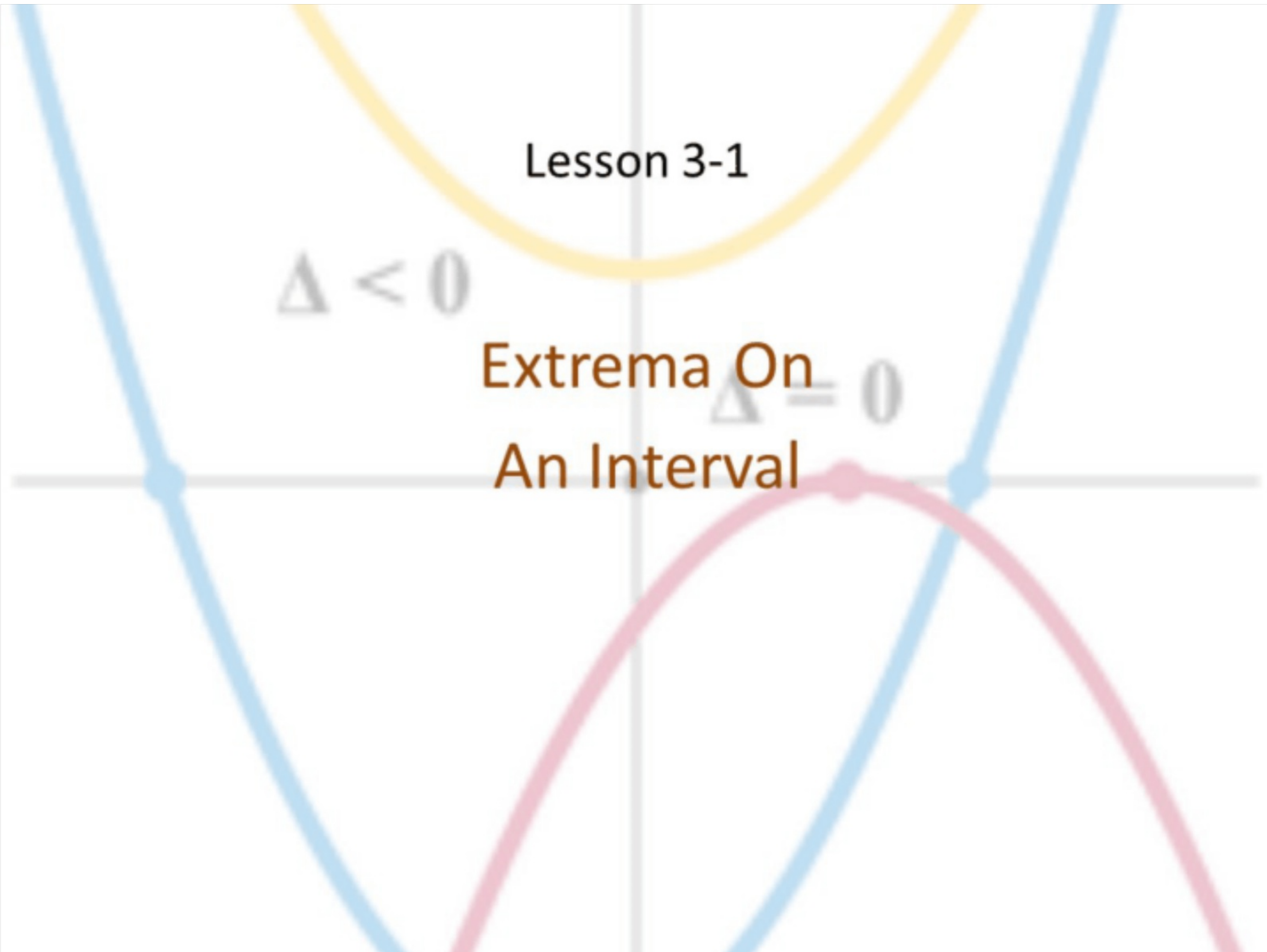
Lesson 3-1

Extrema On An Interval



Objective

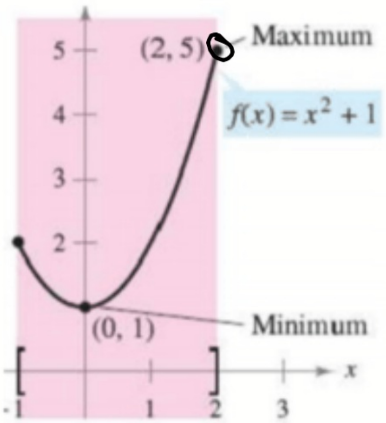
Students will…

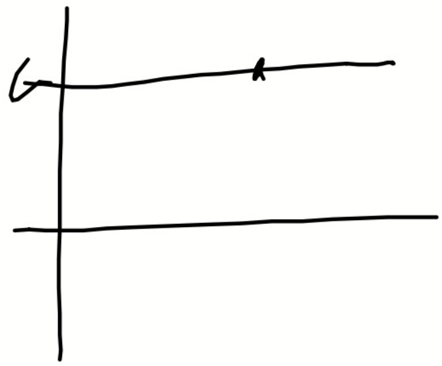
- Be able to understand what an extrema is over an open and closed intervals.
- Be able to distinguish between relative and an absolute extrema.

Extrema

One of the big applications of differentiation is finding an extrema (plural form of the word extremum) of functions. Extrema are the maximum and minimum (extreme) values of a function over an interval.

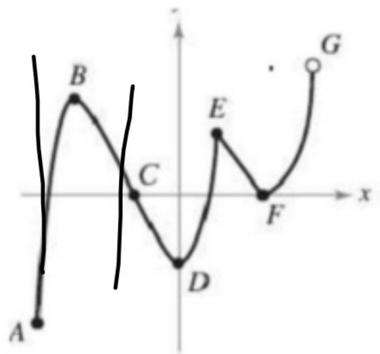
<u>Extreme Value Theorem</u>- If f is continuous on a closed interval $[a, b]$, then f has a both minimum and a maximum on the interval.





Absolute vs Relative Extrema

A great way to distinguish absolute and relative extrema is to consider whether the interval is open or closed.



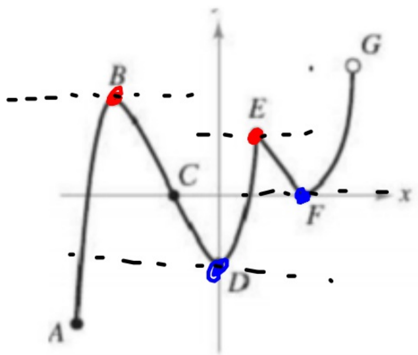
On an open interval (A, G), there are no single extreme values.

On a closed interval $[A, G]$, however, there are single extreme values.

Another good way to identify relative extrema is to think of relative maximum as a hill (or a mountaintop), and relative minimum as a valley.

Application of Differentiation

So, what connection can we make between derivatives and the extrema?



The instantaneous rate of Change at each of the relative min/max is zero.

i.e. relative min/max exist on the x-int. of the derivative function

(*) These points are known as the critical points/values.

Application of Differentiation

Overall, we can formulize the steps in finding the extrema.

GUIDELINES FOR FINDING EXTREMA ON A CLOSED INTERVAL

To find the extrema of a continuous function f on a closed interval $[a, b]$, use the following steps.

1. Find the critical numbers of f in (a, b). Values $\Rightarrow f'(x) = 0$ aka. x-int.
2. Evaluate f at each critical number in (a, b).
3. Evaluate f at each endpoint of $[a, b]$.
4. The least of these values is the minimum. The greatest is the maximum.

For open intervals, only do steps 1 and 2 (to find relative extrema).

Examples

$[-1, 2]$.

Find the extrema of $f(x) = 3x^4 - 4x^3$ on the interval $[1, 2]$.

$f'(x) = 12x^3 - 12x^2$

CU: $0 = 12x^3 - 12x^2$

$0 = 12x^2(x-1)$

$12x^2 = 0$ $\quad$ $x - 1 = 0$

$\cancel{x = 0}$ $\quad$ $x = 1$

$\cancel{f(0) = 0}$

$f(1) = -1$

$f(2) = 16$

Abs max: $(2, 16)$

Abs. min: $(1, -1)$.

Examples

Find the extrema of $f(x) = 2x - 3x^{\frac{2}{3}}$ on the interval $[-1, 3]$.

$f'(x) = 2 - 2x^{-1/3}$

CV: $0 = 2 - 2x^{-1/3}$

$\frac{-2}{-2} = \frac{-2x^{-1/3}}{-2}$

$1 = x^{-1/3} = \frac{1}{\sqrt[3]{x}}$

$x = 1$

$f(-1) = 2(-1) - 3(-1)^{2/3}$

$= -2 - 3 = -5$

$f(1) = 2(1) - 3(1)^{2/3} = -1$

$f(3) = 2(3) - 3(3)^{2/3} \approx -0.24$

abs. max: $(3, -0.24)$

abs. min: $(-1, -5)$

$2\frac{5}{3}$ $\cos(11\pi/3) = \cos(5\pi/3)$ $3\frac{2}{3}$

$\cos(7\pi/3) = \cos(\pi/3)$ $2\frac{1}{3}\pi$

Example

Find the extrema of $f(x) = 2\sin x - \cos 2x$ on the interval $[0, 2\pi]$.

$f'(x) = 2\cos x + 2\sin 2x$

CV: $0 = \cancel{2}(\cos x + \sin 2x)$

$0 = \cos x + \sin 2x$

$0 = \cos x + 2\sin x \cos x$

$0 = \cos x(1 + 2\sin x) \rightarrow \sin x = -1/2$

$x = \pi/2,\ 3\pi/2,\ \frac{7\pi}{6},\ \frac{11\pi}{6}$

$f(0) = -1$

$f(\pi/2) = 3$

$f(7\pi/6) = -3/2$

$f(3\pi/2) = -1$

$f(11\pi/6) = -3/2$

$f(2\pi) = -1$

abs max: $(\pi/2, 3)$

abs min: $(7\pi/6, -3/2)$

$(\frac{11\pi}{6}, -3/2)$.

Homework Due 10/1

TB 3.1- #1-2, 3-11 (odd), 13-35 (odd), 37, 39