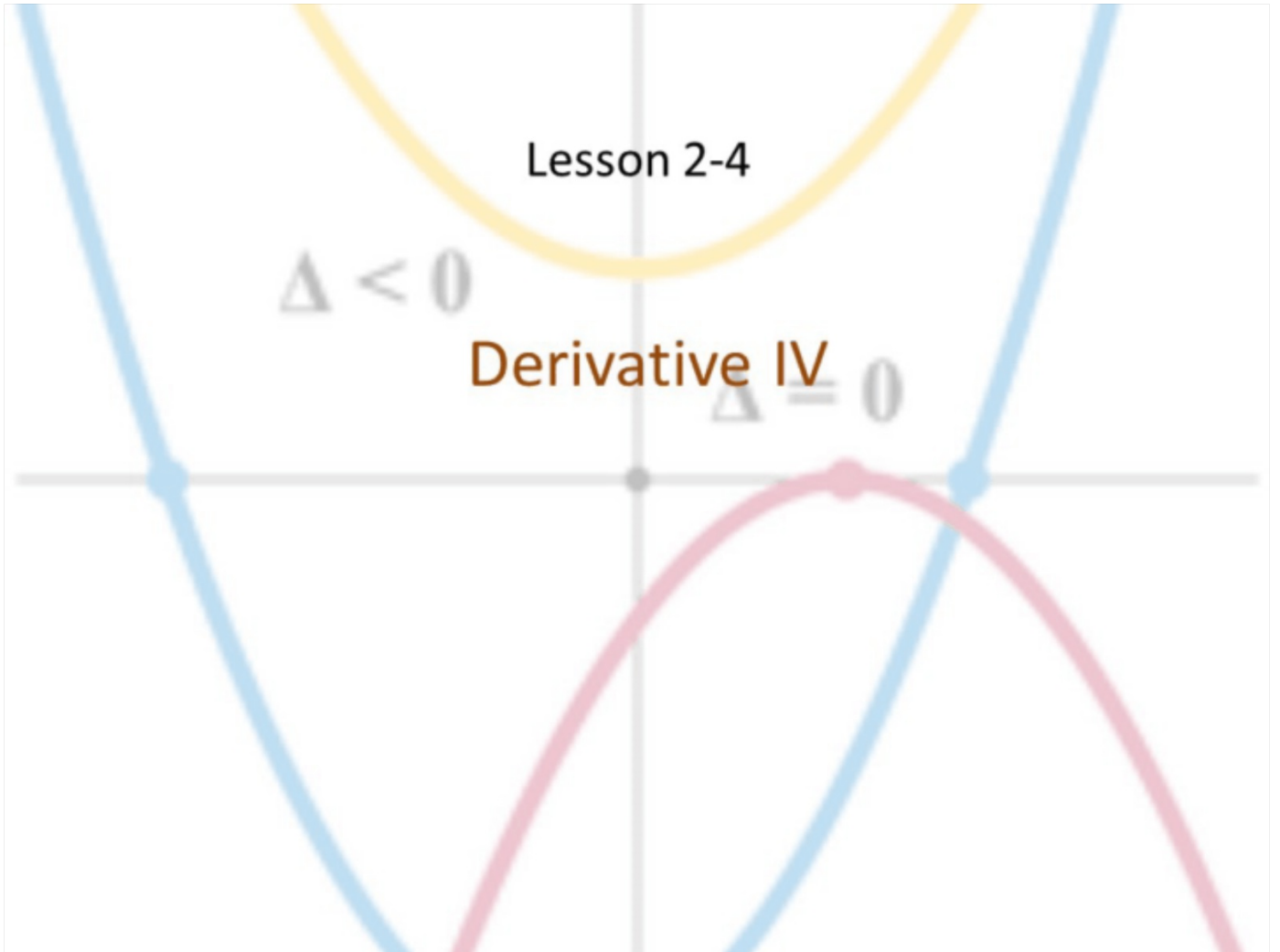


Lesson 2-4

$\Delta < 0$

Derivative IV  $\Delta = 0$



## Objective

Students will...

- Be able to use the chain rule.

## Derivative of Composite Functions

Recall that a composition of functions, or the **composite** function  $f \circ g$  (also called a **composition** of  $f$  and  $g$ ) is defined by

$$(f \circ g)(x) = f(g(x))$$

For finding the derivative of any composite function, we need to use the **chain rule**.

## Chain Rule

**Chain Rule**- If  $f$  and  $g$  are differentiable functions, then the composite function  $f \circ g = f(g(x))$  is also differentiable such that...

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

In other words, the chain rule consists of first taking the derivative of the “outer” function, while leaving the “inner” function the same, then multiplied by the derivative of the “inner” function.

The most challenging part of the chain rule is being able to distinguish the different functions that are being composed (i.e. “outer” and “inner”).

## Examples

Parse the "outer" and the "inner" functions of the following:

a.  $y = \frac{1}{x+1}$

out:  $\frac{1}{x} = x^{-1}$

in:  $x+1$

b.  $y = \sin 2x$

out:  $\sin x$

in:  $2x$

c.  $y = \sqrt{3x^2 - x + 1}$

out:  $\sqrt{x} = x^{1/2}$

in:  $3x^2 - x + 1$

d.  $\tan^2 x = (\tan x)^2$

out:  $x^2$

in:  $\tan x$

## Example

Find the derivative.

a.  $y = (x^2 + 1)^3$

$$y' = 3(x^2 + 1)^2 \cdot 2x$$
$$= \boxed{6x(x^2 + 1)^2}$$

### Example

Find the derivative.

$$\text{b. } f(x) = \sqrt[3]{(x^2 - 1)^2} = ((x^2 - 1)^2)^{1/3} = (x^2 - 1)^{2/3}$$

$$f'(x) = \frac{2}{3}(x^2 - 1)^{-1/3} \cdot 2x$$

$$= \frac{4}{3}x(x^2 - 1)^{-1/3}$$

## Example

Find the derivative.

$$c. g(x) = -\frac{7}{(2t-3)^2} = -7(2t-3)^{-2}$$

$$g'(x) = 14(2t-3)^{-3} \cdot 2$$
$$= \boxed{28(2t-3)^{-3}}$$



Find the derivative.

d.  $g(x) = \cos 2x$

$$g'(x) = -\sin 2x \cdot 2$$
$$= -2 \sin 2x$$

Example  $f'g + g'f$

e.  $y = 3x \sin 2x$

$$y' = 3 \sin 2x + 3x \cos 2x \cdot 2$$
$$= 3 \sin 2x + 6x \cos 2x$$

## Example

Find the derivative.

$$f. y = \sin^3 4t = (\sin 4t)^3$$

$$y' = 3 \sin^2 4t \cdot \cos 4t \cdot 4$$

$$= 12 \sin^2 4t \cos 4t$$

Example

Find the derivative.  $\frac{f'g - g'f}{g^2}$

g.  $\frac{\tan^3(3-t)}{11x^2}$

$$= \frac{(\tan^2(3-t) \cdot \sec^2(3-t) \cdot -1)(11x^2) - 22x(\tan^3(3-t))}{121x^4}$$

$$= \frac{33x^2 \tan^2(3-t) \sec^2(3-t) - 22x \tan^3(3-t)}{121x^4}$$

Slope of the  
Example

Find an equation of the tangent line to the graph of...

$$f(x) = 2 \sin x + \cos 2x$$

$$f'(x) = 2 \cos x - \sin 2x \cdot 2$$

$$= 2 \cos x - 2 \sin 2x$$

## Homework Due 9/13

TB 2.4- #1-6, 7-35 (e.o.o), 45-65 (e.o.o), 80, 82, 91, 95

ex.  $\sin^2\left(\frac{3x}{x-3}\right)$