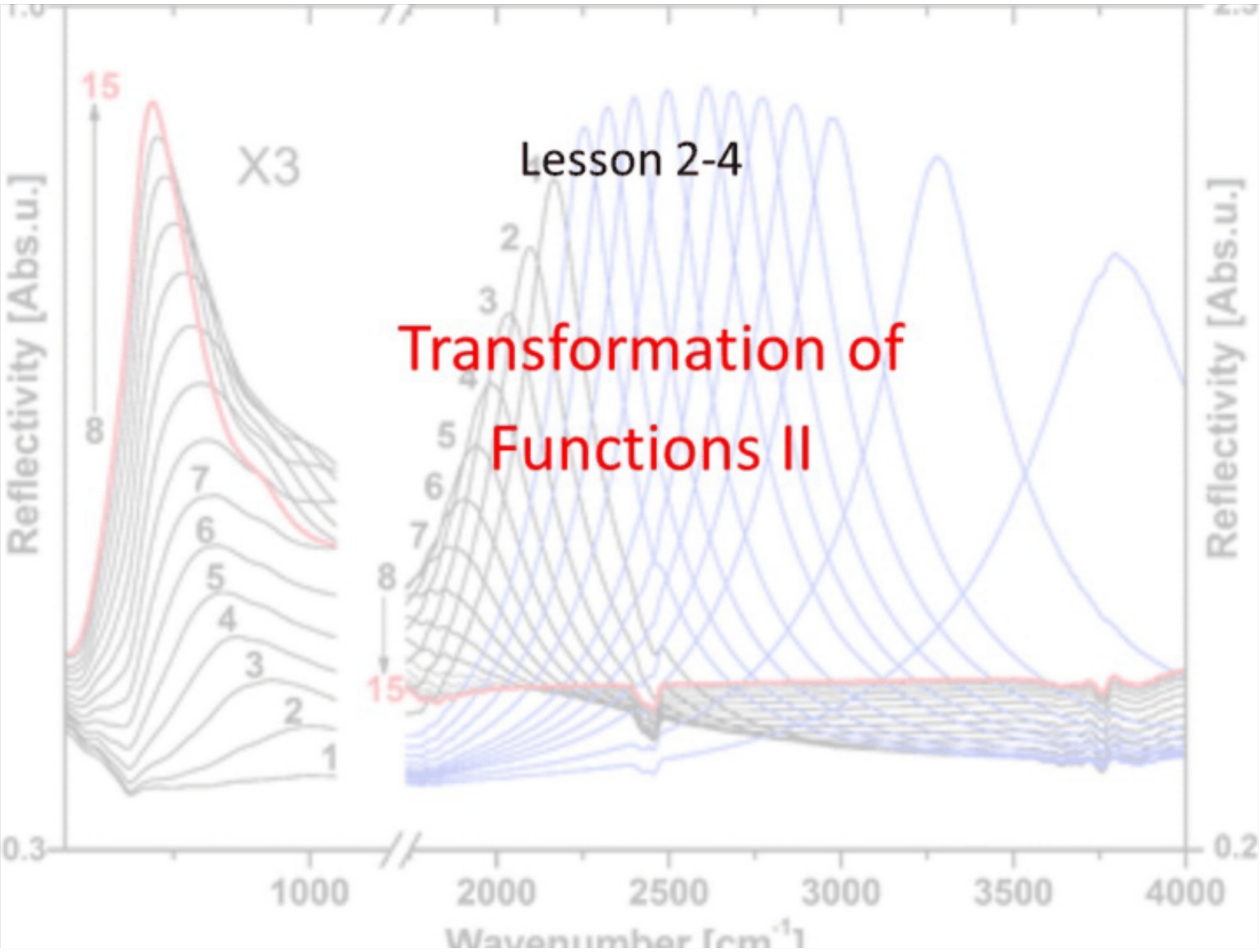
Lesson 2-4

Transformation of Functions II



Objective

Students will…

- Be able to apply the properties of <u>reflections</u> in graphing various functions.
- Be able to apply the properties of <u>stretch and compression</u> in graphing various functions.
- Be able to determine the scale factor of the stretch or compression.

Transformation: Reflection

Second major transformation of functions is **reflection**. Reflection is like a mirror image, and it can either be a **horizontal or vertical** reflection. This can be generalized by the following:

Along the y-axis (horizontal)

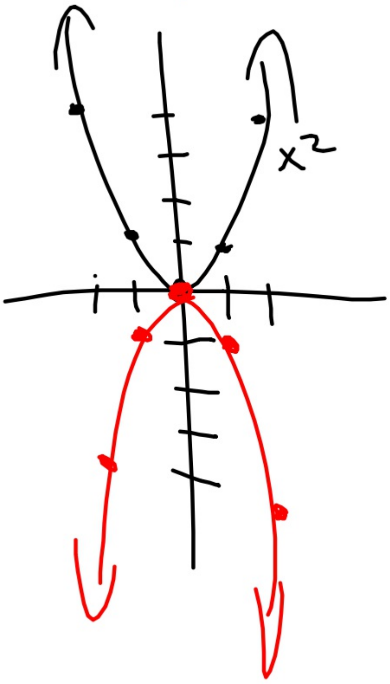
> $y = f(-x)$ reflects the graph of $y = f(x)$ along the y-axis (horizontal reflection).

Along the x-axis (vertical)

> $y = -f(x)$ reflects the graph of $y = f(x)$ along the x-axis (vertical reflection).

Transformation of Functions

Let's go ahead and compare the two functions: $f(x) = x^2$ and $g(x) = -x^2$ Vertical reflection



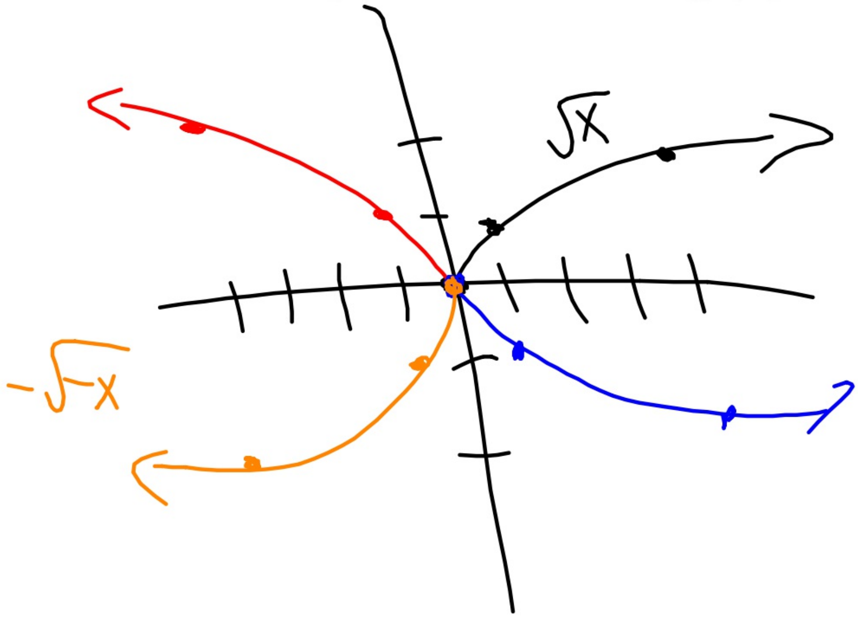
horiz ref.

$$g(x) = (-x)^2 = x^2$$

Transformation of Functions

Now let's compare the functions: $f(x) = \sqrt{x}$ and $g(x) = \sqrt{-x}$

horiz reflection



$-\sqrt{x}$

Examples

Sketch the following functions by transforming its "parent" function.

a. $f(x) = -|x|$

Vert. reflection

b. $f(x) = (-x)^3$

horiz reflection

Transformation: Stretch and Compression

As observed, the transformation that took place was a vertical **stretch or a compression** by a certain **scale factor**. This can be generalized by the following:

\# that multiplies.

For $y = cf(x)$

If $c > 1$, stretch the graph of $y = f(x)$ vertically by a factor of c.

If $0 < c < 1$, compress the graph of $y = f(x)$ vertically by a factor of c.

$\frac{3}{4}$ vs $\frac{4}{3}$

<1 >1

Transformation: Stretch and Compression

As observed, the transformation that took place was a horizontal **stretch or a compression** by a certain **scale factor**. This can be generalized by the following:

For $y = f(cx)$

If $c > 1$, compress the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{c}$

If $0 < c < 1$, stretch the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{c}$

Note the **opposite relationship** of the scale factor between vertical and horizontal stretch/compression.

Transformation of Functions

Describe the transformation given its parent function.

$f(x) = x^2$ and $g(x) = (2x)^2$

horizontal compression by $\frac{1}{2}$.

$g(x) = (2x)^2 = 4x^2$

Vertical stretch by 4

Transformation of Functions

Describe the transformation given its parent function.

$f(x) = x^2$ and $g(x) = \left(\frac{1}{2}x\right)^2$

horizontal stretch by 2.

$g(x) = \left(\frac{1}{2}x\right)^2 = \frac{1}{4}x^2$

Vertical compression by 1/4.

Examples

Determine whether the function has a vertical or a horizontal stretch/compression, and determine its scale factor.

a. $f(x) = 3x^2$

Vertical Stretch by 3

b. $f(x) = \left(\frac{1}{2}x\right)^3$

c. $h(x) = \frac{3}{4}(x-1)^{19}$

d. $p(x) = \sqrt{3x}$

horiz. Compression by $\frac{1}{3}$

e. $f(x) = \frac{5}{4}|x|$

f. $q(x) = \frac{8}{5}\sqrt[6]{x-1}$

g. $u(x) = \frac{10}{11}(x-990)^5$

h. $t(x) = 3\sqrt{\frac{7}{6}(x+5)}$

Examples

For the function given function f, write the equation for the final transformed graph, based on the description of the transformation done.

$f(x) = \sqrt[3]{x}$; shift 3 units to the left, stretch vertically by a factor of 5, and reflect ~~in~~ the x-axis.

over

$-5\sqrt[3]{x+3}$

Homework Due 8/28

Describing and Writing Transformation of Functions WKSHT II