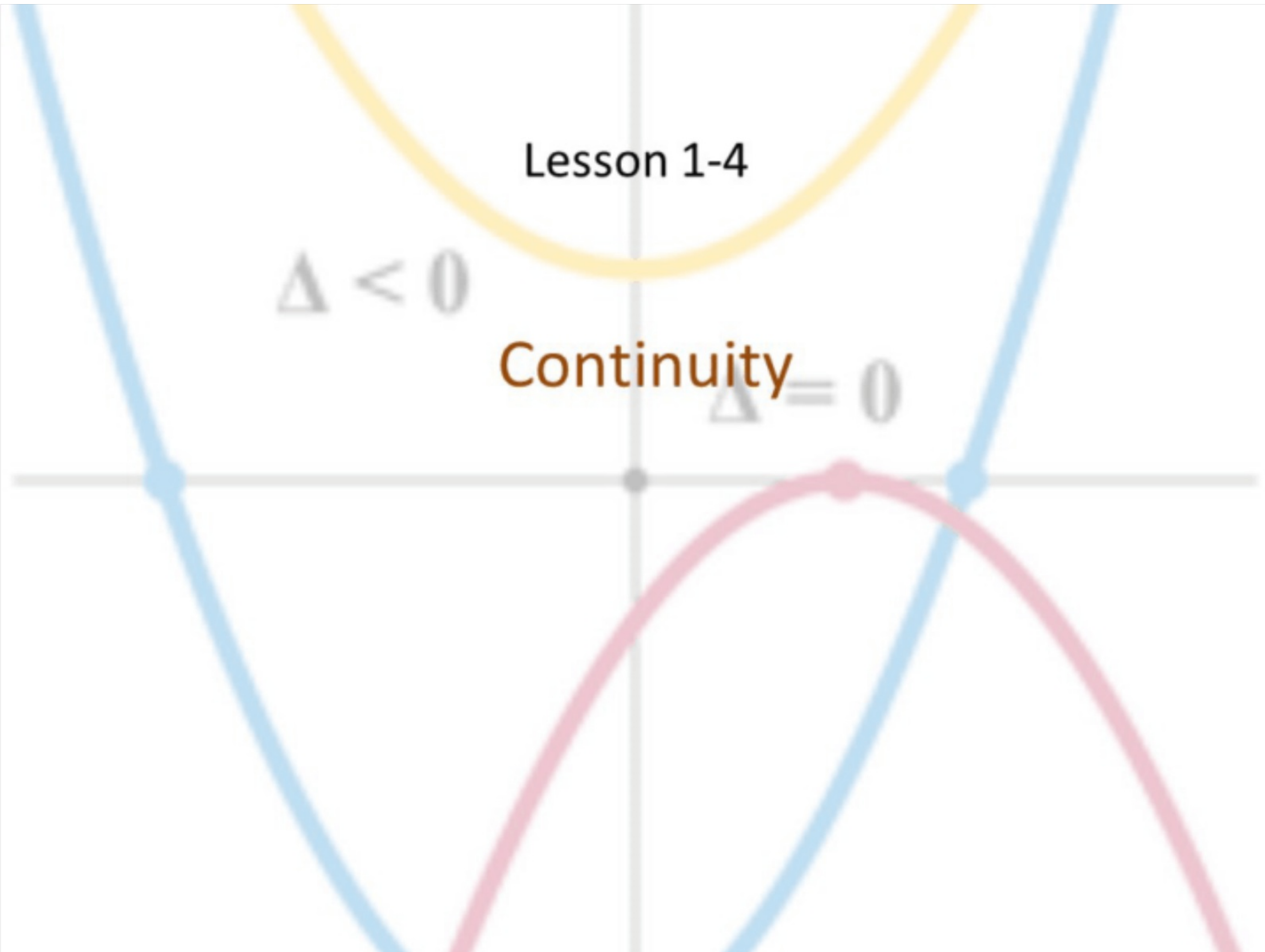
Lesson 1-4

Continuity



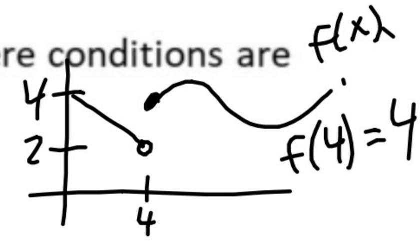
Objective

Students will…

- Be able to distinguish between removable and nonremovable discontinuities.
- Be able to define and use the intermediate value theorem.

Continuity

A function, say f, is **continuous at c** when these there conditions are met:



1. $f(c)$ is defined (i.e. can be evaluated)
2. $\lim_{x \to c} f(x)$ exists. (left side limit = right side limit)
3. $\lim_{x \to c} f(x) = f(c)$

Recall: We can show that a function has a limit at any given point by the existence of limit theorem:

$\lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x)$, (the right and the left side limits are equal)

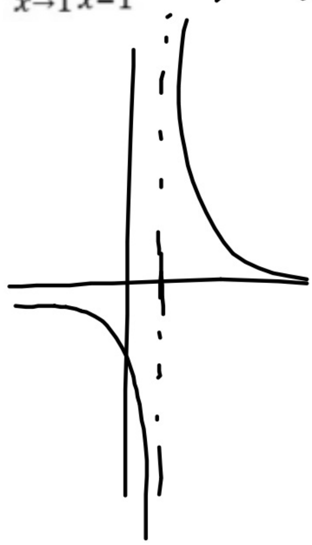
Types of Discontinuity

Always remember that not all discontinuities are created equal! In fact, just because a discontinuity exists at a certain point, this doesn't automatically indicate that the limit doesn't exist. Consider the following problems:

a. $\lim\limits_{x \to 1} \left(\frac{x^2-1}{x-1} = \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} = (x+1)\right)$

$= \boxed{2}$

b. $\lim\limits_{x \to 1} \frac{1}{x-1} = \text{DNE}.$



Types of Discontinuity

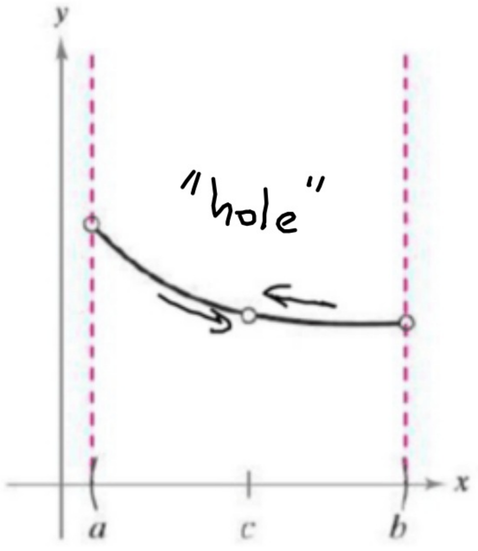
Clearly, (a) has a limit, while (b) did not. Algebraically speaking, simple factoring and simplifying allowed us to find the limit for (a), while there was nothing that could have been done to find a limit for (b). This can be more easily seen looking at their graphs.

In general…

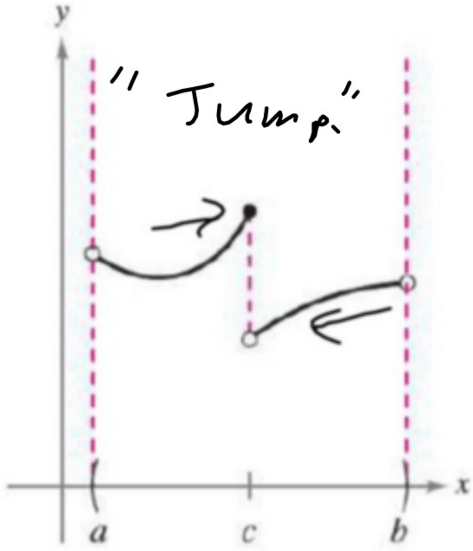
If the limit exists at a certain point of a function, say c, while the function is undefined at c, then the function is said to have a **removable discontinuity** at c.

If the limit does not exist at c, nor is defined at c, then the function is said to have a **nonremovable discontinuity** at c.

Removable vs Nonremovable Discontinuity



(a) Removable discontinuity



(b) Nonremovable discontinuity

Example

Find points of discontinuity, and determine if they are removable or nonremovable discontinuity(ies).

a. $f(x) = \frac{4}{x-6}$

$x - 6 = 0$

$x = 6$

Non removable.

b. $f(x) = \frac{x-5}{x^2-25}$

$= \frac{\cancel{x-5}}{(x+5)\cancel{(x-5)}}$

$= \frac{1}{x+5}$

$x^2 - 25 = 0$

$\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$.

Removable @ $x = 5$, $\lim = \frac{1}{10}$

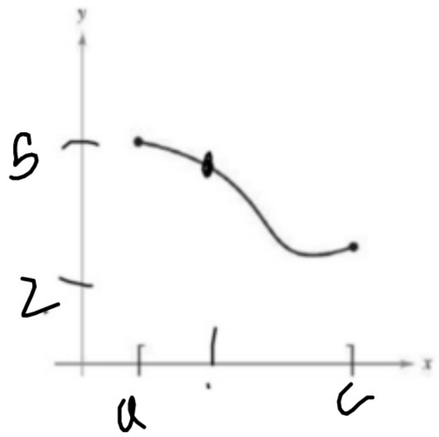
Non-Removable @ $x = -5$, Lim DNE.

Intermediate Value Theorem

There is a very simple but important theorem in Calculus regarding continuity.

<u>Intermediate Value Theorem(IVT)</u>- If f is continuous on the closed interval $[a, c]$, and $f(a) \neq f(c)$, and k is any number between $f(a)$ and $f(c)$, then there is at least one number b in $[a, c]$ such that $f(b) = k$.

In other words, in the interval $[a, c]$, if $a \leq b \leq c$, then $f(b)$ exists, such that $f(a) \leq f(b) \leq f(c)$.



Example

Use the Intermediate Value Theorem to show that the polynomial function $f(x) = x^3 + 2x - 1$ has a zero (x-intercept or root) in the interval $[0, 1]$.

$f(0) = -1$

$f(1) = 2$

By IVT, since $f(0) < 0$ and, $f(1) > 0$, there must be some value, c, in the interval $[0, 1]$, such that $f(c) = 0$.

$\tan\frac{\pi}{6} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}}$
$= \frac{1}{\sqrt{3}}$

Example

Use the Intermediate Value Theorem to show that the function

$f(x) = -\frac{5}{x} + \tan\frac{\pi x}{10}$ has a zero (x-intercept or root) in the interval $[1, 4]$.

$f(1) = -\frac{5}{1} + \tan\frac{\pi}{10} < 0$

$f(4) = -\frac{5}{4} + \tan\frac{2\pi}{5} > 0$

By IVT, since $f(1) < 0$
$f(4) > 0$,

there must be some value, c, in the interval [1,4], such that $f(c) = 0$.

Homework 8/27

TB 1.4 #7-23 (e.o.o), 25-28, 31, 33-53 (e.o.o), 83, 86