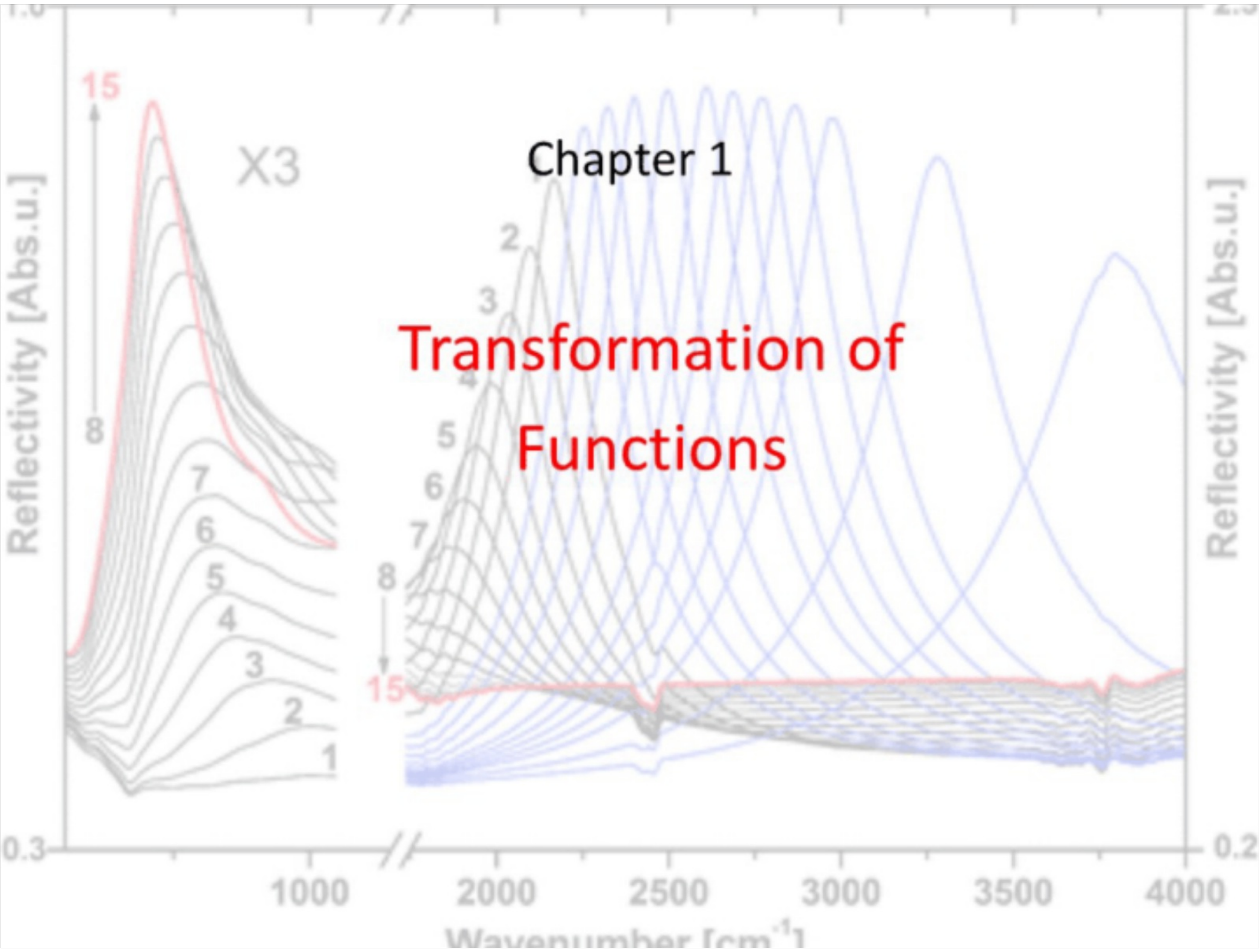
Chapter 1

Transformation of Functions



Objective

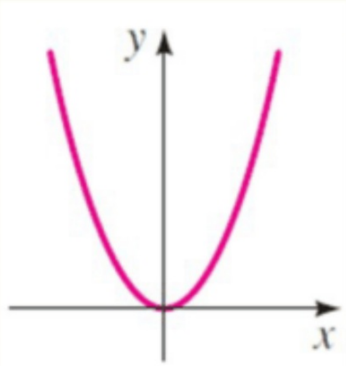
Students will…

- Be able to understand the basic idea of transformation of functions.
- Be able to see and understand the basic functional structures.
- Explore and apply the properties of vertical and horizontal shifts.

"Parent" Functions

We have seen and studied some of the standard functions and their graphs. For example,

$f(x) = x^2$



$f(x) = x^2$

$f(x) = \sqrt{x}$

$f(x) = |x|$

$f(x) = \frac{1}{x}$

$f(x) = a^x$ where $a > 0$, $a \neq 1$

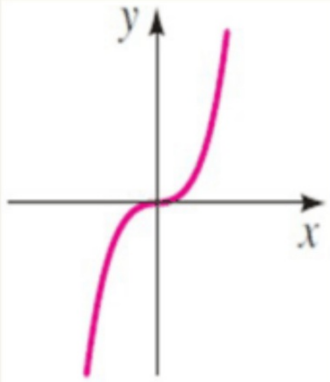
Trig.

$f(\theta) = \sin\theta$

$f(\theta) = \cos\theta$

$f(\theta) = \tan\theta$.

$f(x) = x^3$



$f(x) = x^3$

Functional Structures

Much of mathematics is seeing and understanding (algebraic) structures. This is especially true when it comes to graphing functions. There are two basic components of functional structures:

1. "Inside" vs "Outside"

Ex. $f(x) = (x-1)^2 + 3$ $\qquad$ $g(x) = 3\sqrt{x+2} - 1$

$f(x) = x^2$ $\qquad$ $g(x) = \sqrt{x}$

2. The four basic mathematical operations: $+, -, \times, \div$

Inside vs Outside

Differentiate and describe the inside and outside operations (if any) of the following functions.

a. $f(x) = (x-1)^3 - 1$

Subtract 1 → inside

subtract 1 → outside.

Parent: $f(x) = x^3$

b. $g(x) = 2\sqrt{x} - 1$

multiply 2 → outside.

Subtract 1 → outside.

Parent: $f(x) = \sqrt{x}$

c. $h(x) = 2^{(x+1)} - 3$

add 1 → inside.

Subtract 3 → outside.

Parent: $f(x) = 2^x$

d. $f(x) = -\frac{1}{(x+1)} - 3$

Parent $f(x) = \frac{1}{x}$

add 1 → inside.

multiply −1 → outside.

subtract 3 → outside

Transformation Guidelines

Here are the basic guidelines of transformation of functions:

1. The operations occurring on the "inside" of the function equation always affects the graph **horizontally** (left or right).

2. The operations occurring on the "inside" must be treated inversely (opposite operation).

Ex. For "+" think or treat it as "−" and vice-versa. (×↔÷)

3. The operations occurring on the "outside" must be treated "as is" (not opposite), and it always affects the graph **vertically** (up or down).

4. There are **three** basic transformations: shift, stretch/compress, and flip (translation, dilation, and reflection).

Transformation: Vertical Shift

~~As observed, the difference between $f(x)$ and $g(x)$ was that $g(x)$ was simply $f(x)$ vertically~~ ~~**shifted up 2 units**~~. This can be generalized by the following:

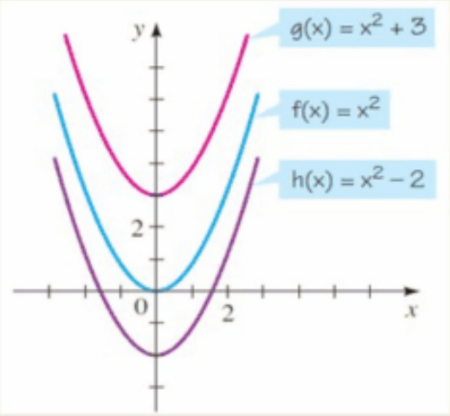
$y = f(x) \pm c$ shifts the graph of $y = f(x)$ upward(+) or downward(-) c units, for $c > 0$.

Ex. Use the graph of $f(x) = x^2$ to sketch the graph of,

$g(x) = x^2 + 3$ and $h(x) = x^2 - 2$

up 3

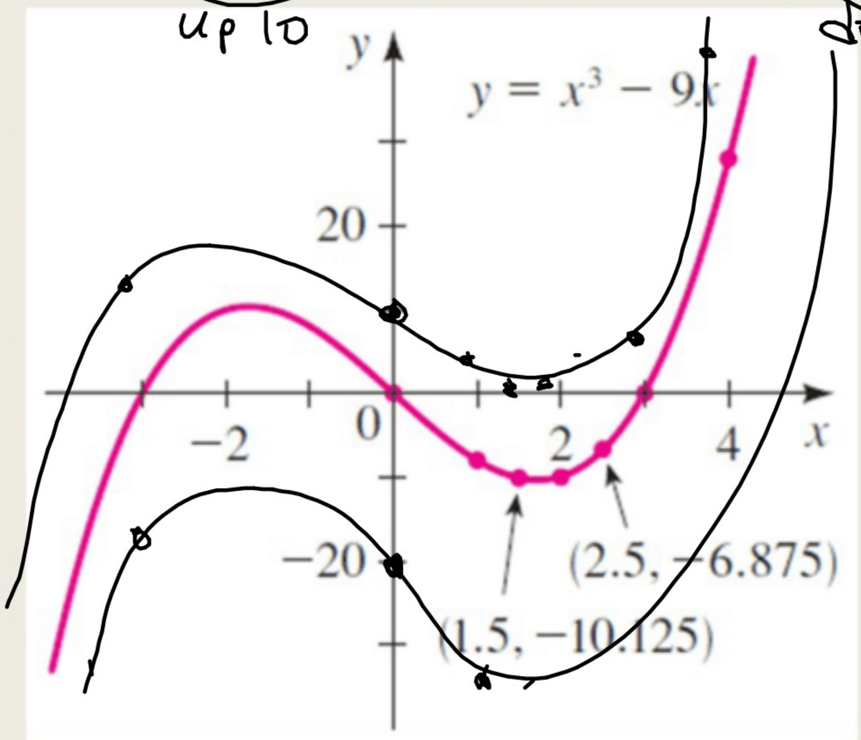
down 2



Example

Use the graph of $f(x) = x^3 - 9x$ shown below to sketch the graph of $g(x) = x^3 - 9x + 10$ and $h(x) = x^3 - 9x - 20$

up 10 down 20



Transformation: Horizontal Shift

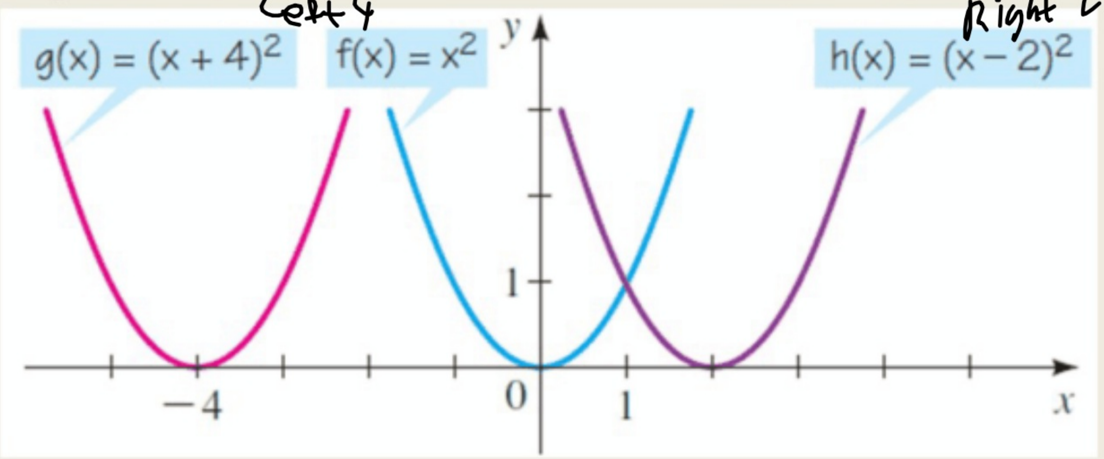
So the horizontal shift can also be generalized.

$y = f(x \pm c)$ shifts the graph of $y = f(x)$ to the right(−) or left(+) c units, for $c > 0$. Note the **opposite** signs!

Ex. Use the graph of $f(x) = x^2$ to sketch the graph of,

$g(x) = (x + 4)^2$ and $h(x) = (x - 2)^2$

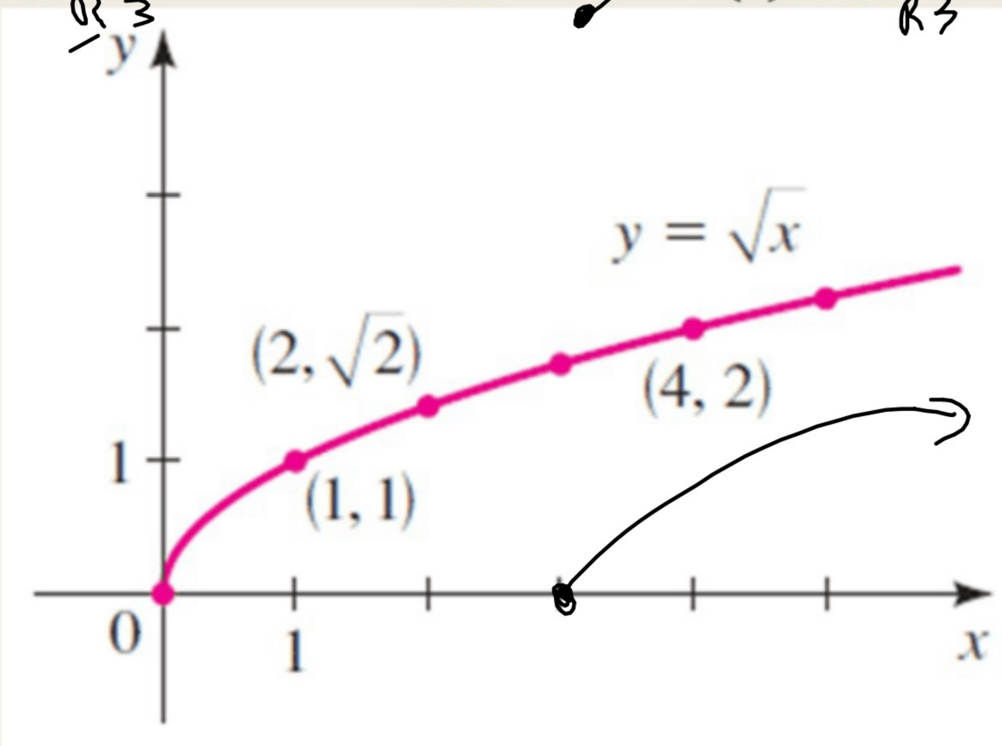
Left 4 Right 2



Example

Use the graph of $f(x) = \sqrt{x}$ shown below to sketch the graph of
$g(x) = \sqrt{x-3}$ and $h(x) = \sqrt{x-3} + 4$

R 3 R3 U4



Examples

Describe the shift of the function: $g(x) = (x + 11)^2 - 2$ from its "parent" function, $f(x) = x^2$

L11, D2

Describe the shift of the function $h(x) = \sin(\theta - \pi) + 3$ from its "parent" function, $f(x) = \sin\theta$

Rπ, U3

Describe the shift of the function $p(x) = \frac{1}{x+2} - 1$ from its "parent" function, $f(x) = \frac{1}{x}$

L2, D1.

Homework Due 8/27

Describing and Writing Transformation of Functions WKSHT