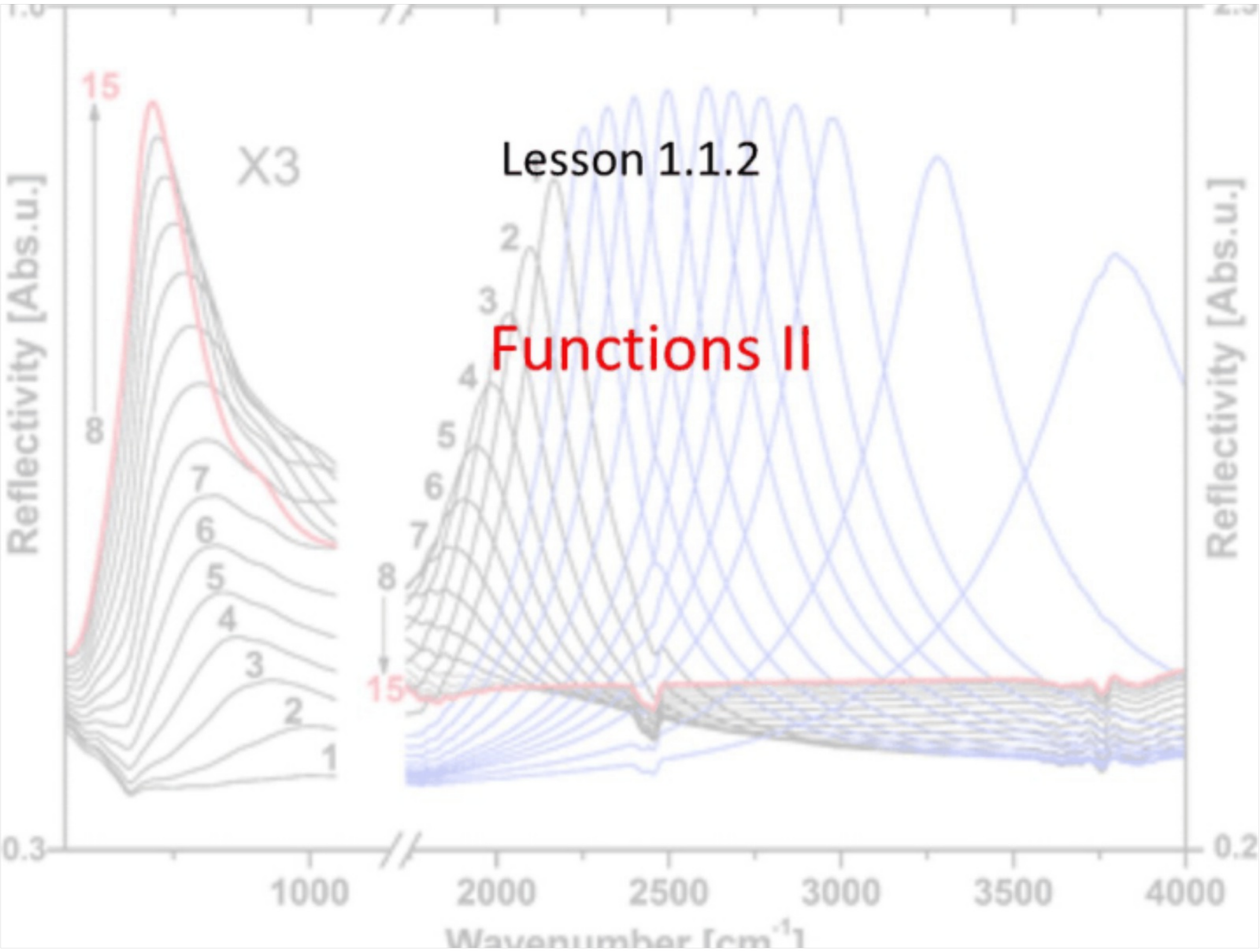
Lesson 1.1.2

Functions II



Objective

Students will…

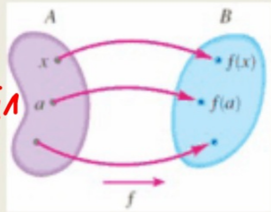
- Be able to use and write interval notations.
- Be able to identify and represent the domain of a function algebraically.
- Be able to identify and present the domain and range of a function graphically.

Definition of a Function

So now we are ready to define what a function is.

A **function**, say f, is a rule that assigns to each element (item) x in a certain set A **exactly one** element, called $f(x)$, in a set B.

In

Ex. 

Domain (x.)

Range (f(x) or y)

Out

Another way to define function is for every **input**, there is exactly **one** **output**.

The set A is also known as the **domain**, and set B is known as the **range**.

Interval Notation

One of the best ways to represent an interval of numbers is using the **interval notation**. Interval notation has two components:

1. "[,]" means closed, while "(,)" means open.
2. Closed intervals include the last (or the "edge") numbers (think min and/or max), and open intervals do not, while including all the numbers leading up to it.

Ex. $[1,2]$ represents the interval $1 \leq x \leq 2$

Ex. $(1,2)$ represents the interval $1 < x < 2$

Ex. $[1, 2)$ represents the interval $1 \leq x < 2$

Note: $\pm$Infinity is always **open**.

Domain of a Function

A **domain** of a function is the set of all **inputs**. Domain may be written **explicitly**. For example, for the function $f(x) = x^2, \quad 0 \le x \le 5$, the domain is specifically set as all inputs between and including 0 and 5. Hence its domain is simply $[0, 5]$. (Piecewise Function)

Whenever we have a function without the domain stated explicitly, we need to figure it out by algebraic reasoning.

Domain of a Function

Here are some guidelines for finding the domain of a function.

1. If a function is a polynomial, then the domain is always all real numbers, or $(-\infty, \infty)$.
   ex. $\sqrt{\ }$, $\sqrt[4]{\ }$, $\sqrt[6]{\ }$ ...
2. Functions to look out for: even root functions, rational (fraction) functions, and logarithms. Outside of these functions, the domain is almost always $(-\infty, \infty)$.
3. Many times, it may be easier to think what numbers cannot be in the domain to find the domain.

ex $\sqrt{x}$ $\quad x \geq 0$

ex. $\log_a(x)$ $\quad x > 0$.

ex. $\dfrac{x+1}{x}$ $\quad x \neq 0$

Examples

Find the domain.

$f(x) = x^2 + 1$

D: $(-\infty, \infty)$

$g(x) = \frac{1}{x-4}$

D: $x - 4 \neq 0$

$x \neq 4$

$(-\infty, 4) \cup (4, \infty)$

$h(x) = \sqrt{x}$

D: $x \geq 0$

$[0, \infty)$

Examples

Find the domain of each function.

a. $f(x) = \frac{1}{x^2 - x}$

b. $g(x) = \sqrt{9 - x^2}$

$9 - x^2 \geq 0$

$+x^2 \quad +x^2$

$\pm 3 = \sqrt{9} \geq \sqrt{x^2}$

$\boxed{\begin{matrix} x \leq 3 \\ x \geq -3 \end{matrix}}$

c. $h(t) = \frac{t}{\sqrt{t+1}}$

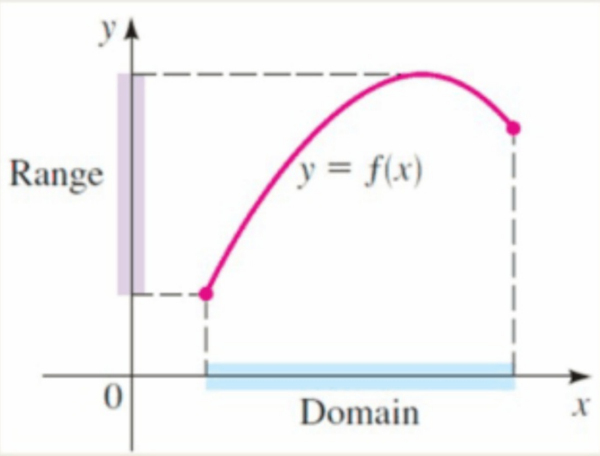
$t + 1 > 0$ $\quad (-1, \infty)$

$t > -1$

Domain and Range from Graphs

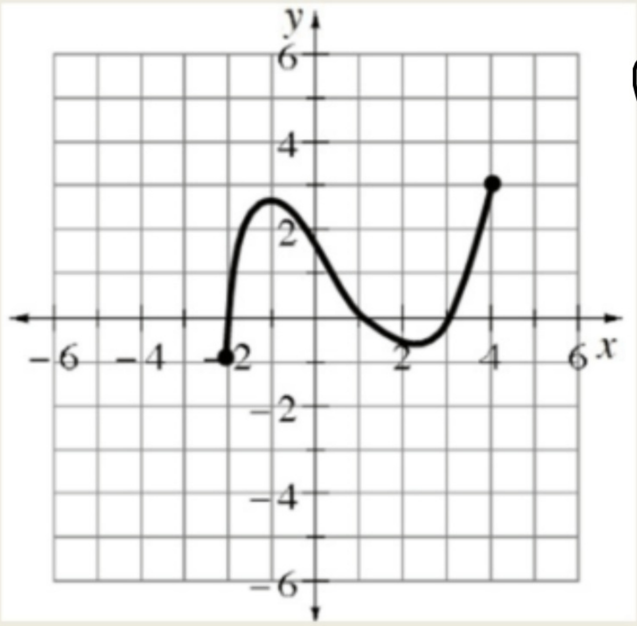
You can also determine the **domain** and the **range** of functions from their graphs. Remember that domain is all possible x-values, while the range is the all possible y-values. So, from the graph the domain is always from the lowest x-coordinates to the highest x-coordinates. Likewise, the range is from the **lowest y-coordinates** to the highest **y-coordinates**.

Ex.



Examples

Identify the domain and range.

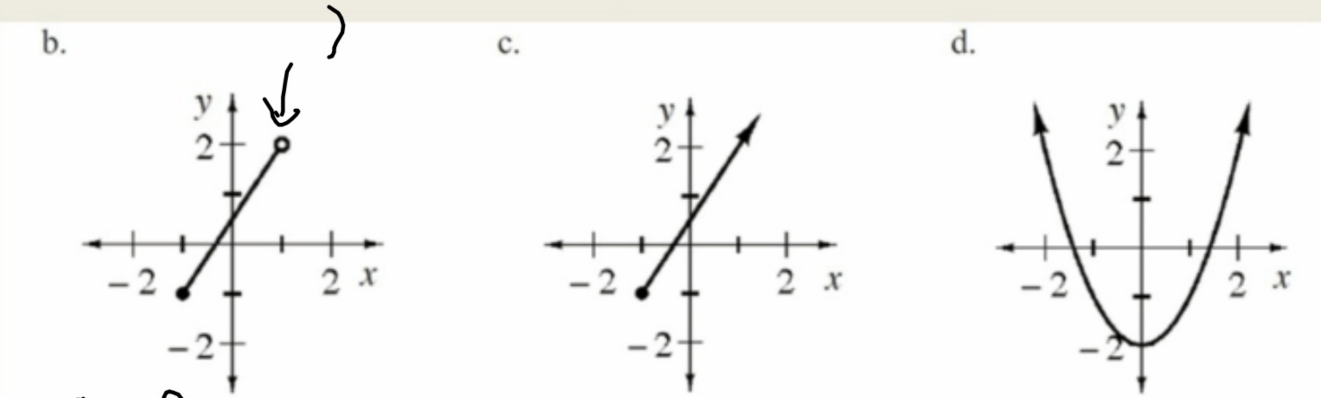


D: [-2, 4]

R: [-1, 3]

Examples

Identify the domain and range.



D: [−1, 1)

R: [−1, 2)

Homework Due 8/16

Domain and Range WKSHT