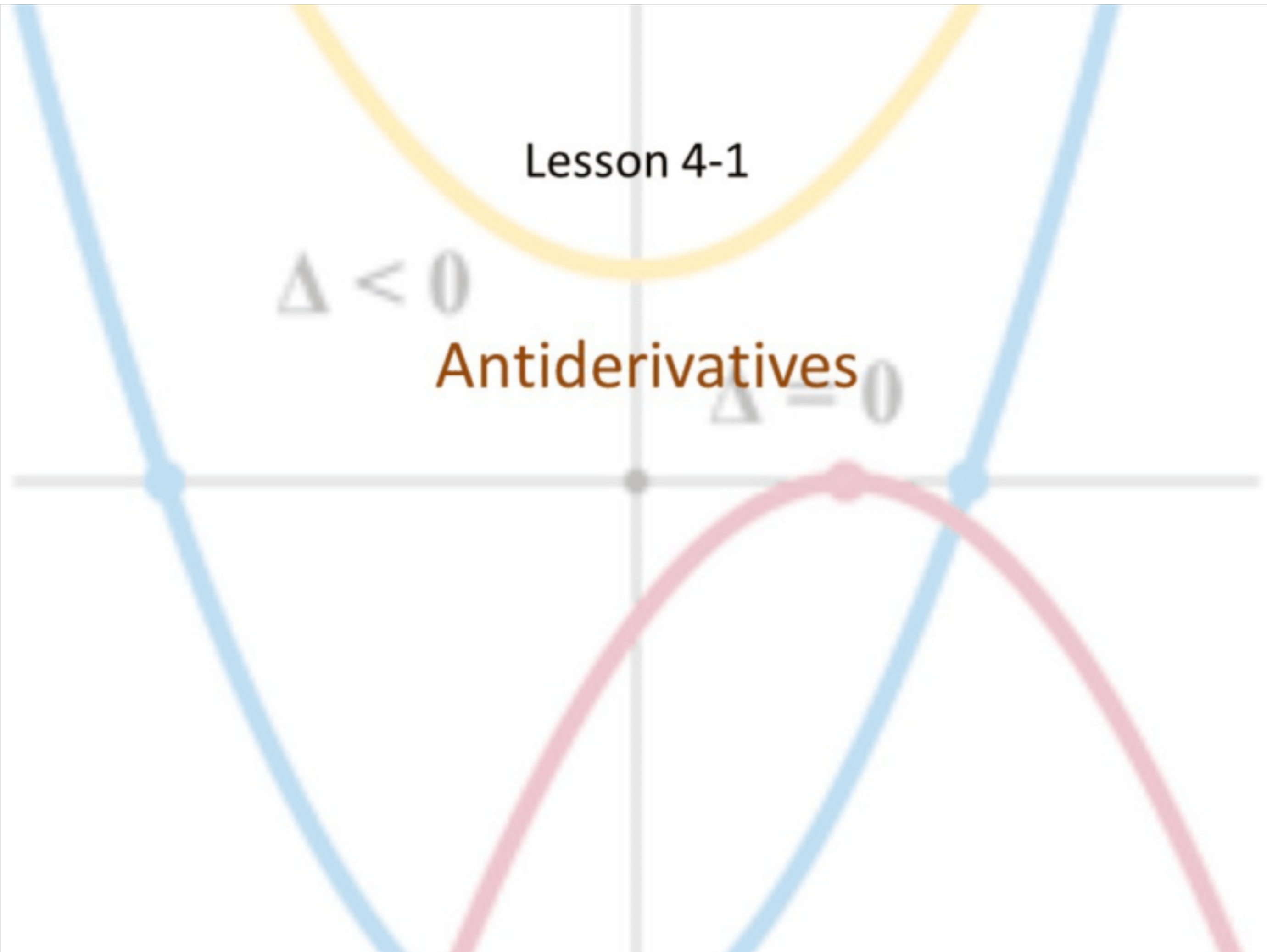

Lesson 4-1
$\Delta < 0$
Antiderivatives
$\Delta = 0$

Objective

Students will…

- Be able to find the antiderivatives.
- Be able to come up with general and particular solutions to differential equations.

Antiderivatives

One of the key components in mathematics is being able to revert a process. So, naturally, if we can take the derivatives of a function, we should be able to "undo" it. This is what **antiderivatives** are.

Antiderivative- A function $\boxed{F}$ is an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

$\rightarrow F'(x) = 3x^2 = f(x)$.

Ex. If $f(x) = 3x^2$, then possibly, $F(x) = x^3$

Note: Notice the word "possibly," because there are almost always multiple antiderivatives. From the above example,

If $f(x) = 3x^2$, then it is also possible that $F(x) = x^3 - 89$

The "Anti-Power Rule"

The Power Rule is probably the easiest and the simplest derivative rule. The antiderivatives involving the Power Rule is also quite simple.

$\rightarrow F(x) = \frac{3}{3}x^3 = \boxed{x^3}$

Consider…

Ex. $f(x) = 3x^2$

$f'(x) = 2 \cdot 3x^{2-1} = 6x^1$

$f(x) = x^2$

$F(x) = \frac{1}{3}x^3$

Thus, the "Anti-Power Rule" is as follows:

If $f(x) = cx^a$, then $F(x) = \boxed{\frac{c}{a+1}x^{a+1}}$

Example

Find the antiderivatives of the following:

a. $f(x) = x^2$

$\boxed{F(x) = \frac{1}{3}x^3 + C}$

b. $f(x) = 5x^3 - 8x^2 + 9$

$\boxed{F(x) = \frac{5}{4}x^4 - \frac{8}{3}x^3 + 9x + C}$

$\frac{dy}{dx} = f(x)$

$dy = f(x)\,dx.$

Differential Equations

Finding the antiderivatives can be presented in multiple ways. One of the ways is differential equations. Note that the **solution to a differential equation is its antiderivative**.

Ex. Find the general solution of the differential equation $y' = 2$

$y = 2x + C$

Example

$\to 1x^{-2}$

Find the particular solution of $f(x) = \frac{1}{x^2}$, with $x > 0$ and the initial condition $F(\overset{x}{1}) = \overset{y}{0}$

$F(x) = \frac{1}{-1}x^{-1} = -x^{-1} = -\frac{1}{x} + C$ general sol.

$0 = -\frac{1}{1} + C$

$1 = C$

$\boxed{F(x) = -\frac{1}{x} + 1}$, $x > 0$.

Particular sol.

Position $= y$

Velocity $= \frac{dy}{dx} = y'$

acceleration $= \frac{d^2y}{dx^2} = y''$

$\boxed{s(t) = -16t^2 + v_0 t + s_0.}$

Vertical Motion Problem

A ball is thrown upward with an initial velocity of 64 feet per second ($t=0$) from an initial height of 80 feet ($t=0$). $(s'' = -32 \text{ ft})$

a. Find the position function giving the height s as a function of the time t.

$s''(t) = -32$

$s'(t) = -32t + C$

$s'(0) = 64 = -32(0) + C$

$64 = C$

$s'(t) = -32t + 64$

$s(t) = \frac{-32}{2}t^2 + 64t + C$

$= -16t^2 + 64t + C$

$s(0) = 80 = -16(0)^2 + 64(0) + C$

$80 = C$

$\boxed{s(t) = -16t^2 + 64t + 80}$

Vertical Motion Problem

b. When does the ball hit the ground?

$$-16t^2+64t+80=0$$

$$\cancel{-16}(t^2-4t-5)=0$$

$$(t+1)(t-5)=0$$

$$t=\cancel{-1},\ \boxed{5}\text{ sec.}$$

Homework Due 11/5

TB 4.1 #55-63 (odd), 67-75 (odd), 81, 82, 87-92