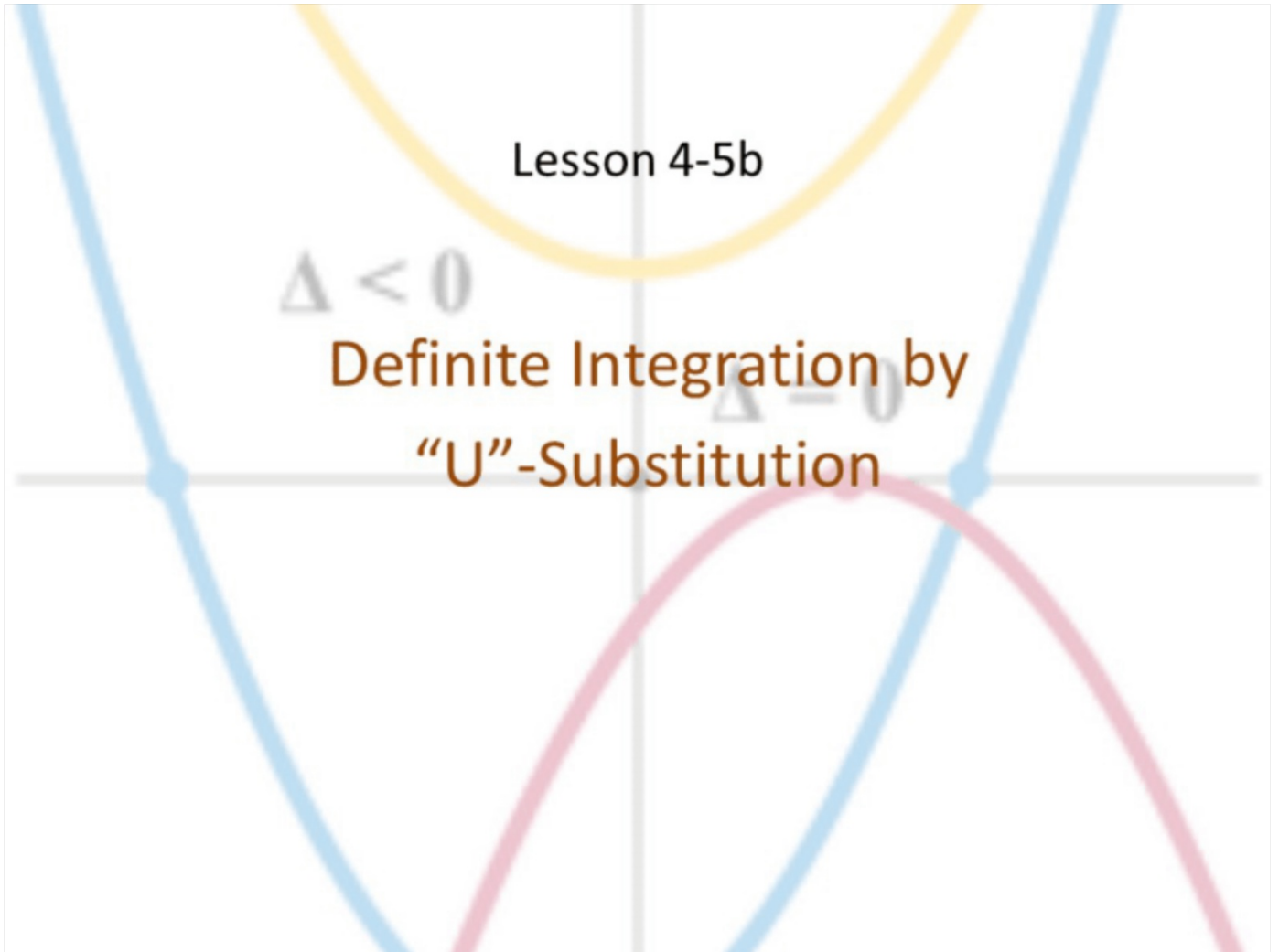


Lesson 4-5b

$\Delta < 0$

Definite Integration by
"U"-Substitution

$\Delta = 0$



Objective

Students will...

- Be able to use “U”-Substitution to calculate definite integrals.
- Be able to use a graphing calculator to find definite integrals.

Definite Integration by U-Substitution

The way to integrate definite integrals follows the same logic, just with a few extra steps. There are two ways: direct way, and using a change of variables. *(bounds)*

Direct way: Rewrite “u” back to the original and use the original bounds.

Change of variable: Change the bounds using “u” and apply the first FTC without rewriting.

Example: Direct Way

Find $\int_0^1 x(x^2 + 3)^3 dx$

$$u = x^2 + 3$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$\Rightarrow \frac{1}{2} \int_0^1 u^3 du = \frac{1}{2} \left(\frac{1}{4} u^4 \right) \Big|_0^1$$

$$= \frac{1}{2} \left(\frac{1}{4} (x^2 + 3)^4 \right) \Big|_0^1$$

$$= \frac{1}{8} (1^2 + 3)^4 - \frac{1}{8} (0^2 + 3)^4$$

$$= \frac{256}{8} - \frac{81}{8} = \boxed{\frac{175}{8}}$$

$$u = x^2 + 3$$

Example: Change of Variable

Find $\int_0^1 x(x^2 + 3)^3 dx$

$$= \frac{1}{8} u^4 \Big|_3^4 = \frac{1}{8}(4)^4 - \frac{1}{8}(3)^4 = \frac{256}{8} - \frac{81}{8} = \boxed{\frac{175}{8}}$$

Example

Find $\int_1^5 \frac{\boxed{x}}{\sqrt{2x-1}} dx$

$$u = 2x - 1 \Rightarrow x = \frac{u+1}{2} = \frac{1}{2}u + \frac{1}{2}$$

$$du = 2 dx$$

$$\frac{1}{2} du = dx$$

$$u_1 = 2(1) - 1 = 1$$

$$u_5 = 2(5) - 1 = 9$$

$$\Rightarrow \frac{1}{2} \int_1^9 \frac{\frac{1}{2}u + \frac{1}{2}}{u^{1/2}} du = \frac{1}{2} \int_1^9 \frac{u'}{2u^{1/2}} + \frac{1}{2u^{1/2}} du$$

$$\Rightarrow \frac{1}{2} \int_1^9 \frac{u^{1/2}}{2} + \frac{u^{-1/2}}{2} du = \frac{1}{4} \int_1^9 u^{1/2} + u^{-1/2} du$$

$$= \frac{1}{4} \left(\frac{2}{3} u^{3/2} + 2u^{1/2} \Big|_1^9 \right) = \frac{1}{4} \left(\frac{2}{3}(9)^{3/2} + 2(9)^{1/2} - \left(\frac{2}{3}(1)^{3/2} + 2(1)^{1/2} \right) \right)$$

$$= \frac{1}{4} \left(18 + 6 - \frac{2}{3} - 2 \right) = \frac{1}{4} \left(\frac{64}{3} \right) = \frac{16}{3}$$

Using a Calculator to Integrate

For the most part, you can only use a graphing calculator to find definite integrals.

Find $\int_0^1 x(x^2 + 3)^3 dx$

Find $\int_1^5 \frac{x}{\sqrt{2x-1}} dx$

Homework Due 11/19

4.5 #71-81(odd), 87-91(odd), 93-98
(no need to graph)