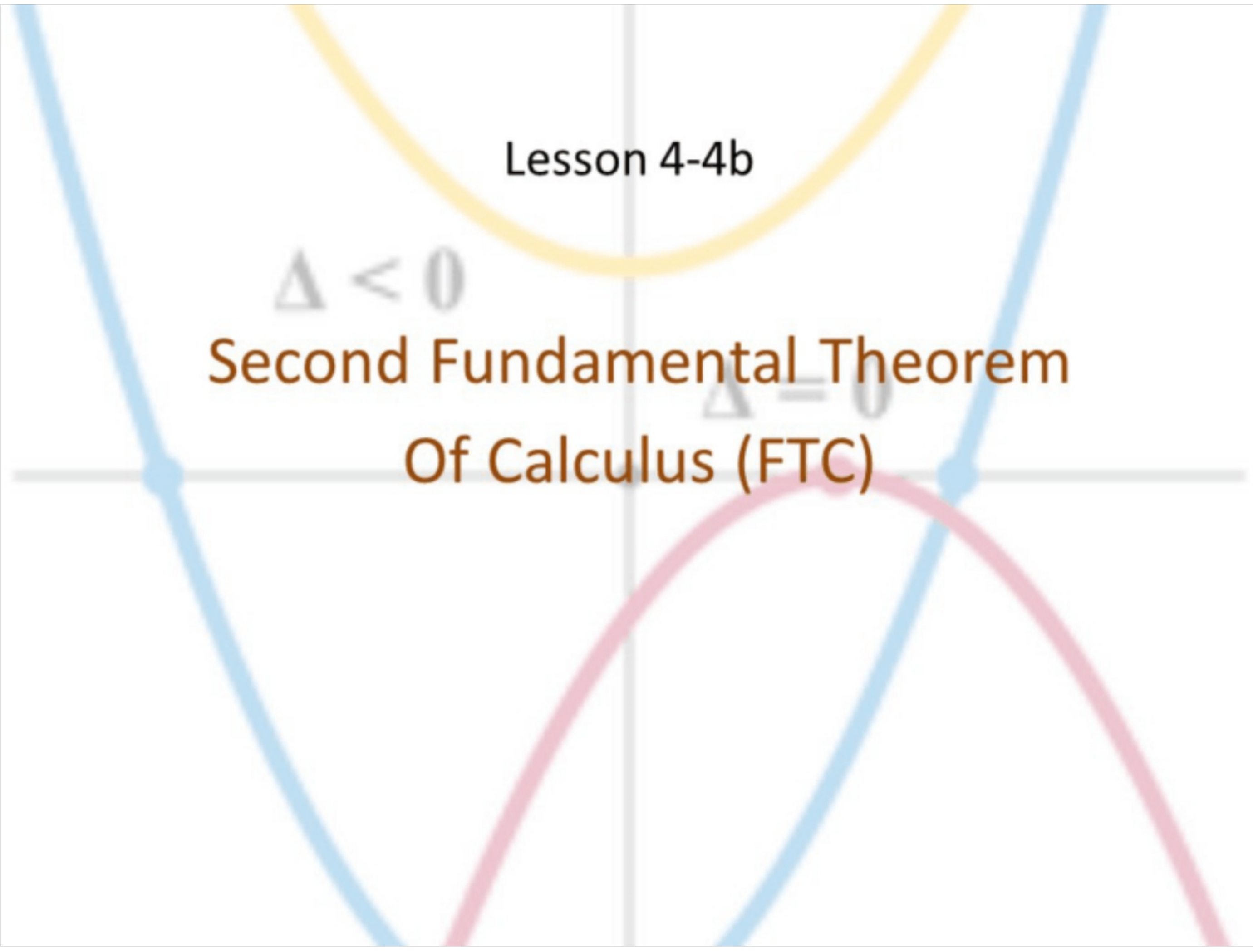
Lesson 4-4b

Second Fundamental Theorem Of Calculus (FTC)



Objective

Students will…

- Be able to know the Second Fundamental Theorem of Calculus (FTC).
- Be able to use the Second FTC to inverse operate between derivatives and definite integrals.

Fundamental Theorem of Calculus

We now finally arrive at the most important (hence fundamental!) theorem of Calculus.

Fundamental Theorem of Calculus- If a function f is continuous on the closed interval $[a, b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$.

Recall that $F(x)$ is the antiderivative of $f(x)$.

The power of this theorem is that we no longer have to resort to approximations, but rather, evaluate the area underneath the curve directly.

$x + \cancel{2} = 3$
$-\cancel{2}$

Derivatives VS Antiderivatives

On the surface it is obvious to see how derivatives (differentiation) and antiderivatives (integration) are inverses of each other. This is readily seen with indefinite integrals. For example…

$$\sin x = \cancel{\int \frac{d}{dx}} \sin x = \cancel{\frac{d}{dx} \int} \sin x$$

$= \int \cos x = \frac{d}{dx}(-\cos x) = \sin x.$

But, what about definite integrals? Consider…

$\frac{d}{dx}\left[\int_a^b f(x)\right] = \frac{d}{dx}\left(F(b) - F(a)\right) = f(b) - f(a) = 0 \Leftrightarrow a, b \in \mathbb{R}$

"constant"

$\frac{d}{dx}\left[\int_a^u f(x)\right] = \frac{d}{dx}\left(F(u) - F(a)\right) = \boxed{f(u) \cdot u' - \cancel{f(a)}}$

* u is not a constant

Second Fundamental Theorem of Calculus

With this we arrive at the following second part to the FTC.

Second Fundamental Theorem of Calculus- If f is continuous on an open interval I, containing a, then for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = (f(x))\left(\frac{d}{dx}x\right)$$

Note: This theorem is irrelevant if the bounds are constants (i.e. real numbers). This trivial case would simply yield 0, because the derivative of any constant is 0.

Example

Evaluate $\frac{d}{dx}\left[\int_0^x \sqrt{t^2+1}\,dt\right]$

$= \sqrt{x^2+1} \cdot \frac{d}{dx} x$

$= \boxed{\sqrt{x^2+1}}$

Example

Evalute $\frac{d}{dx}\left[\int_0^{x^7} \sin t\, dt\right]$

$= \sin(x^7) \cdot \frac{d}{dx}(x^7)$

$= \boxed{7x^6 \sin(x^7)}$

Homework Due 11/13

4.4 Exercises #5-21 (e.o.o), 39-41(odd), 81-86