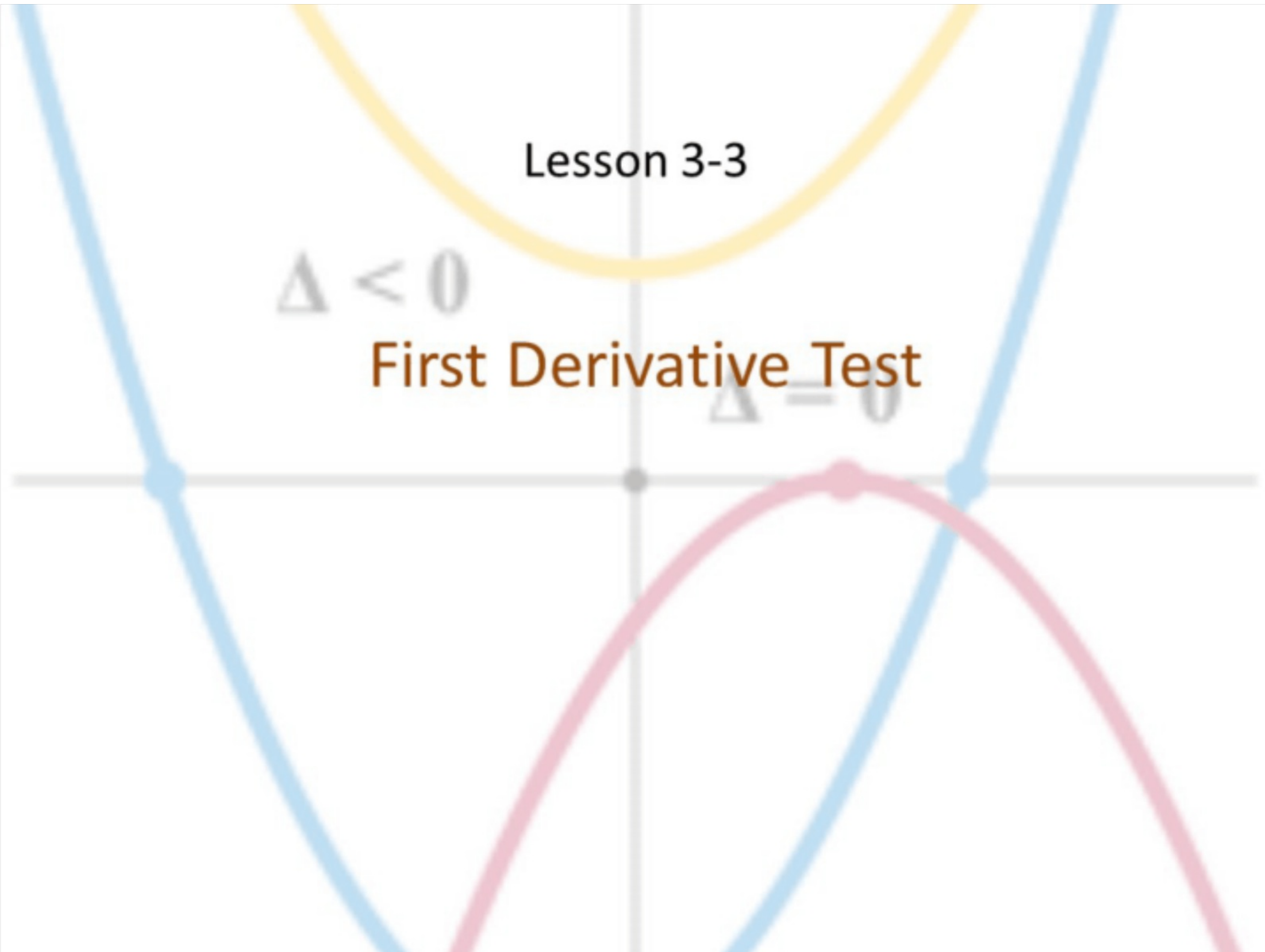

Lesson 3-3
$\Delta < 0$
First Derivative Test
$\Delta = 0$

Objective

Students will…

- Be able to determine intervals on which a function is increasing or decreasing.
- Be able to apply the First Derivative Test to find relative extrema of a function.

Increasing vs Decreasing

Recall from the past that…

A function f is increasing on an interval if for any two numbers a and b in the interval, $a < b$ implies $f(a) < f(b)$. (Pos. Corr.) or (Direct)

A function f is decreasing on an interval if for any two numbers a and b in the interval, $a < b$ implies $f(a) > f(b)$. (Neg. Corr.), or (Inverse).

In other words, moving from **left to right**, if the graph is going up it is increasing, while if it goes down it is decreasing.

Derivatives and Inc/Dec

Considering that the derivative of a function is the equation that finds the rate of change of a function, we have this trivial result...

Inst.

Theorem 3.5- Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). Then...

1. If $f'(x) > 0$, i.e. **positive**, for all x in (a, b), then f is **increasing** on $[a, b]$.
2. If $f'(x) < 0$, i.e. **negative**, for all x in (a, b), then f is **decreasing** on $[a, b]$.
3. If $f'(x) = 0$, for all x in (a, b), then f is **constant** on $[a, b]$.

Remember, derivative represents the **slope**!

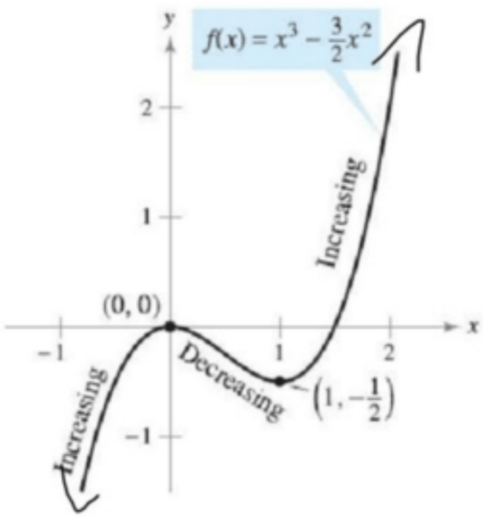
Examples

x-values

Find the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is decreasing or increasing. (Graphically)

Inc: $(-\infty, 0] \cup [1, \infty)$.

Dec: $[0, 1]$.



Example

Find the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is decreasing or increasing. (Algebraically)

$f'(x) = 3x^2 - 3x$

$3x^2 - 3x > 0$

$\underbrace{3x}\underbrace{(x-1)} > 0$

① $3x > 0$ and $x-1 > 0$

~~$x > 0$~~ $x > 1$

② $3x < 0$ and $x-1 < 0$

$x < 0$ ~~$x < 1$~~

$$\boxed{\begin{array}{l} \text{Inc: } (-\infty, 0) \cup (1, \infty). \\ \text{Dec: } (0, 1). \end{array}}$$

$3x^2 - 3x < 0$

$3x(x-1) < 0$

① $3x < 0$ and $x-1 > 0$

~~$x < 0$ $x > 1$~~

② $3x > 0$ and $x-1 < 0$

$x > 0$ $x < 1$

$0 < x < 1$

Example

Find the open intervals on which $y = \frac{x^2}{x+2}$ is decreasing or increasing.

$$y' = \frac{2x(x+2) - x^2}{(x+2)^2} = \frac{2x^2+4x-x^2}{(x+2)^2} = \frac{x^2+4x}{(x+2)^2}$$

Inc: $(-\infty,-4) \cup (0,\infty)$

Dec: $(-4,0)$

$$\frac{x^2+4x}{+(x+2)^2} > 0$$

$x^2+4x > 0$

$x(x+4) > 0$

① $x>0$ and $x+4>0$

$\cancel{x>-4}$

② $\cancel{x<0}$ and $x+4<0$

$x<-4$

$$\frac{x^2+4x}{+(x+2)^2} < 0$$

$x^2+4x < 0$

$x(x+4) < 0$

① $x<0$ and $x+4>0$ $\quad -4<x<0$

$x>-4$

~~② $x>0$ and $x+4<0$~~

~~$x<-4$~~

First Derivative Test

Putting all of this together, we come up with the First Derivative Test, which allows us to find all of the relative minimums and maximums.

(*) Critical values include points of discont.

The First Derivative Test- Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

Dec inc.

1. If $f'(x)$ changes from **negative to positive** at c, then f has a **relative minimum** at $(c, f(c))$.

inc. dec.

2. If $f'(x)$ changes from **positive to negative** at c, then f has a **relative maximum** at $(c, f(c))$.
3. If $f'(x)$ either positive or negative on both sides of c, then $f(c)$ **neither** a relative minimum nor a relative maximum.

Example

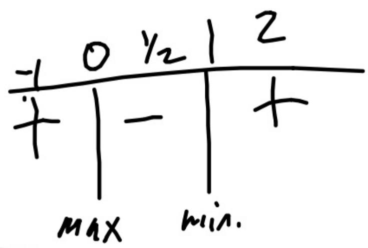
Find the relative extrema of $f(x) = x^3 - \frac{3}{2}x^2$

$f'(x) = 3x^2 - 3x$

CV: $0 = 3x^2 - 3x$

$0 = 3x(x-1)$

$x = 0, 1$



$\boxed{\begin{array}{l}\text{rel max: } (0, 0) \\ \text{rel min: } (1, -1/2)\end{array}}$

Inc: $(-\infty, 0) \cup (1, \infty)$

Dec: $(0, 1)$

Example

Find the relative extrema of $y = \frac{x^2}{x+2}$

$$y' = \frac{x^2+4x}{(x+2)^2}$$

$$0 = \frac{x^2+4x}{(x+2)^2} \quad +$$

$$x^2+4x=0$$

c.v.: $x(x+4)=0$

$$x = 0, -4, -2$$

	-4		-2		0	
-5		-3		-1		1
$+$		$-$		$-$		$+$
	max				min	

$$\boxed{\begin{array}{l}\text{rel max: } (-4,-8) \\ \text{rel min: } (0,0)\end{array}}$$

Example

Find the relative extrema of the function $f(x) = \frac{1}{2}x - \sin x$

$f'(x) = \frac{1}{2} - \cos x$

CV: $0 = \frac{1}{2} - \cos x$

$\frac{1}{2} = \cos x$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\frac{5\pi}{6} + \frac{\sqrt{3}}{2}$

$\frac{1}{2}\left(\frac{\pi}{3}\right) - \frac{\sqrt{3}}{2}$

$\pi/6 - \sqrt{3}/2$

$0 \quad \pi/3 \quad \pi \quad 5\pi/3 \quad 2\pi$

$- \quad + \quad -$

min max.

rel min: $\left(\pi/3, \frac{\pi - 3\sqrt{3}}{6}\right)$

rel max: $\left(5\pi/3, \frac{5\pi + 3\sqrt{3}}{6}\right)$.

Example

$$\frac{1}{\sqrt[3]{x^2-4}}$$

Find the relative extrema of $f(x) = (x^2 - 4)^{\frac{2}{3}}$

$$f'(x) = \frac{2}{3}(x^2-4)^{-1/3} \cdot 2x$$

CV:

$$0 = \frac{4}{3}x(x^2-4)^{-1/3}$$

$\frac{4}{3}x = 0$ $\quad$ $x^2 - 4 = 0$

$x = 0$ $\quad$ $x = \pm 2$

	-2		0		2	
-3		-1		1		3
$-$		$+$		$-$		$+$
	min		max		min.	

rel min: $(-2, 0)$, $(2, 0)$

rel max: $(0, \sqrt[3]{16})$

Example

Find the relative extrema of $f(x) = \frac{x^4+1}{x^2}$

$$f'(x) = \frac{4x^3(x^2) - 2x(x^4+1)}{x^4}$$

$$= \frac{4x^5 - 2x^5 - 2x}{x^4}$$

cv: $0 = \frac{2x^5 - 2x}{x^4}$

$0 = 2x^5 - 2x$

$0 = x(2x^4 - 2)$

$x = 0, \pm 1$

-2	-1	-1/2	0	1/2	1	2
−		+		−		+
	min		max		min	

rel. min: $(-1, 2)$, $(1, 2)$

rel. max: DNE.

$(f(0) = DNE)$

Homework Due 10/26

TB 3.3 #1-8, 9-15 (odd), 17-37 (e.o.o), 39-45 (odd)