1.5 Exercises

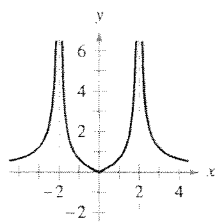
See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–4, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches 4 from the left and from the right.

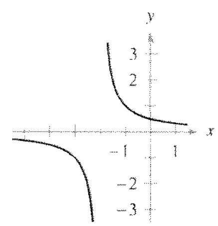
1. $f(x) = \dfrac{1}{x-4}$
2. $f(x) = \dfrac{-1}{x-4}$
3. $f(x) = \dfrac{1}{(x-4)^2}$
4. $f(x) = \dfrac{-1}{(x-4)^2}$

In Exercises 5–8, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -2 from the left and from the right.

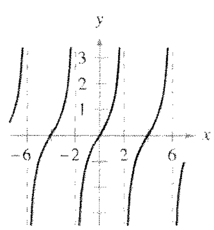
5. $f(x) = 2\left|\dfrac{x}{x^2-4}\right|$



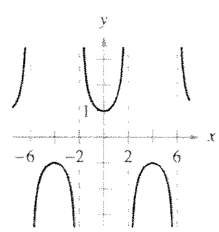
6. $f(x) = \dfrac{1}{x+2}$



7. $f(x) = \tan\dfrac{\pi x}{4}$



8. $f(x) = \sec\dfrac{\pi x}{4}$



Numerical and Graphical Analysis **In Exercises 9–12, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -3 from the left and from the right by completing the table. Use a graphing utility to graph the function to confirm your answer.**

x	-3.5	-3.1	-3.01	-3.001
$f(x)$				

x	-2.999	-2.99	-2.9	-2.5
$f(x)$				

9. $f(x) = \dfrac{1}{x^2-9}$
10. $f(x) = \dfrac{x}{x^2-9}$
11. $f(x) = \dfrac{x^2}{x^2-9}$
12. $f(x) = \sec\dfrac{\pi x}{6}$

In Exercises 13–32, find the vertical asymptotes (if any) of the graph of the function.

13. $f(x) = \dfrac{1}{x^2}$
14. $f(x) = \dfrac{4}{(x-2)^3}$
15. $f(x) = \dfrac{x^2}{x^2-4}$
16. $f(x) = \dfrac{-4x}{x^2+4}$
17. $g(t) = \dfrac{t-1}{t^2+1}$
18. $h(s) = \dfrac{2s-3}{s^2-25}$
19. $h(x) = \dfrac{x^2-2}{x^2-x-2}$
20. $g(x) = \dfrac{2+x}{x^2(1-x)}$
21. $T(t) = 1 - \dfrac{4}{t^2}$
22. $g(x) = \dfrac{\frac{1}{2}x^3 - x^2 - 4x}{3x^2 - 6x - 24}$
23. $f(x) = \dfrac{3}{x^2+x-2}$
24. $f(x) = \dfrac{4x^2+4x-24}{x^4-2x^3-9x^2+18x}$
25. $g(x) = \dfrac{x^3+1}{x+1}$
26. $h(x) = \dfrac{x^2-4}{x^3+2x^2+x+2}$
27. $f(x) = \dfrac{x^2-2x-15}{x^3-5x^2+x-5}$
28. $h(t) = \dfrac{t^2-2t}{t^4-16}$
29. $f(x) = \tan \pi x$
30. $f(x) = \sec \pi x$
31. $s(t) = \dfrac{t}{\sin t}$
32. $g(\theta) = \dfrac{\tan\theta}{\theta}$

In Exercises 33–36, determine whether the graph of the function has a vertical asymptote or a removable discontinuity at $x = -1$. Graph the function using a graphing utility to confirm your answer.

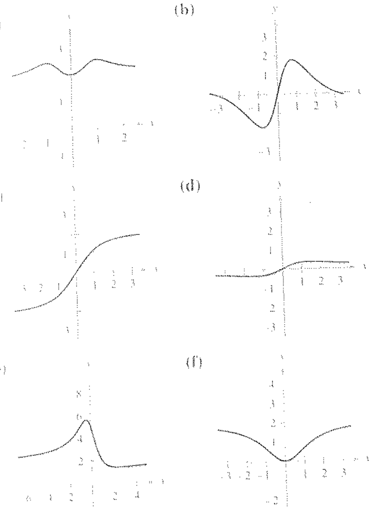
33. $f(x) = \dfrac{x^2-1}{x+1}$
34. $f(x) = \dfrac{x^2-6x-7}{x+1}$
35. $f(x) = \dfrac{x^2+1}{x+1}$
36. $f(x) = \dfrac{\sin(x+1)}{x+1}$

In Exercises 37–54, find the limit (if it exists).

37. $\displaystyle\lim_{x\to -1^+} \frac{1}{x+1}$
38. $\displaystyle\lim_{x\to 1^-} \frac{-1}{(x-1)^2}$
39. $\displaystyle\lim_{x\to 2^+} \frac{x}{x-2}$
40. $\displaystyle\lim_{x\to 1^+} \frac{2+x}{1-x}$
41. $\displaystyle\lim_{x\to 1^+} \frac{x^2}{(x-1)^2}$
42. $\displaystyle\lim_{x\to 4^-} \frac{x^2}{x^2+16}$
43. $\displaystyle\lim_{x\to -3} \frac{x+3}{x^2+x-6}$
44. $\displaystyle\lim_{x\to (-1/2)^+} \frac{6x^2+x-1}{4x^2-4x-3}$
45. $\displaystyle\lim_{x\to 1} \frac{x-1}{(x^2+1)(x-1)}$
46. $\displaystyle\lim_{x\to 3} \frac{x-2}{x^2}$
47. $\displaystyle\lim_{x\to 0^-} \left(1 + \frac{1}{x}\right)$
48. $\displaystyle\lim_{x\to 0} \left(x^2 - \frac{1}{x}\right)$
49. $\displaystyle\lim_{x\to 0^+} \frac{2}{\sin x}$
50. $\displaystyle\lim_{x\to (\pi/2)^+} \frac{-2}{\cos x}$
51. $\displaystyle\lim_{x\to \pi} \frac{\sqrt{x}}{\csc x}$
52. $\displaystyle\lim_{x\to 0} \frac{x+2}{\cot x}$
53. $\displaystyle\lim_{x\to 1/2} x \sec \pi x$
54. $\displaystyle\lim_{x\to 1/2} x^2 \tan \pi x$

1.6 Exercises

Matching In Exercises 1–6, match the function with one of the graphs [(a), (b), (c), (d), (e), or (f)] using horizontal asymptotes as an aid.



1. $f(x) = \dfrac{2x^2}{x^2+2}$ 2. $f(x) = \dfrac{2x}{\sqrt{x^2+2}}$

3. $f(x) = \dfrac{x}{x^2+2}$ 4. $f(x) = 2 + \dfrac{x^2}{x^4+1}$

5. $f(x) = \dfrac{4\sin x}{x^2+1}$ 6. $f(x) = \dfrac{2x^2-3x+5}{x^2+1}$

Numerical and Graphical Analysis In Exercises 7–12, use a graphing utility to complete the table and estimate the limit as x approaches infinity. Then use a graphing utility to graph the function and estimate the limit graphically.

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

7. $f(x) = \dfrac{4x+3}{2x-1}$ 8. $f(x) = \dfrac{2x^2}{x+1}$

9. $f(x) = \dfrac{-6x}{\sqrt{4x^2+5}}$ 10. $f(x) = \dfrac{10}{\sqrt{2x^2-1}}$

11. $f(x) = 5 - \dfrac{1}{x^2+1}$ 12. $f(x) = 4 + \dfrac{3}{x^2+2}$

Finding Limits at Infinity In Exercises 13 and 14, find $\lim_{x\to\infty} h(x)$, if possible.

13. $f(x) = 5x^3 - 3$
 (a) $h(x) = \dfrac{f(x)}{x^2}$
 (b) $h(x) = \dfrac{f(x)}{x^3}$
 (c) $h(x) = \dfrac{f(x)}{x^4}$

14. $f(x) = -4x^2 + 2x - 5$
 (a) $h(x) = \dfrac{f(x)}{x}$
 (b) $h(x) = \dfrac{f(x)}{x^2}$
 (c) $h(x) = \dfrac{f(x)}{x^3}$

Finding Limits at Infinity In Exercises 15–18, find each limit, if possible.

15. (a) $\lim_{x\to\infty} \dfrac{x^2+2}{x^3-1}$
 (b) $\lim_{x\to\infty} \dfrac{x^2+2}{x^2-1}$
 (c) $\lim_{x\to\infty} \dfrac{x^2+2}{x-1}$

16. (a) $\lim_{x\to\infty} \dfrac{3-2x}{3x^3-1}$
 (b) $\lim_{x\to\infty} \dfrac{3-2x}{3x-1}$
 (c) $\lim_{x\to\infty} \dfrac{3-2x^2}{3x-1}$

17. (a) $\lim_{x\to\infty} \dfrac{5-2x^{3/2}}{3x^2-4}$
 (b) $\lim_{x\to\infty} \dfrac{5-2x^{3/2}}{3x^{3/2}-4}$
 (c) $\lim_{x\to\infty} \dfrac{5-2x^{3/2}}{3x-4}$

18. (a) $\lim_{x\to\infty} \dfrac{5x^{3/2}}{4x^2+1}$
 (b) $\lim_{x\to\infty} \dfrac{5x^{3/2}}{4x^{3/2}+1}$
 (c) $\lim_{x\to\infty} \dfrac{5x^{3/2}}{4\sqrt{x}+1}$

Finding a Limit In Exercises 19–42, find the limit.

19. $\lim_{x\to\infty} \left(4 + \dfrac{3}{x}\right)$ 20. $\lim_{x\to-\infty} \left(\dfrac{5}{x} - \dfrac{x}{3}\right)$

21. $\lim_{x\to\infty} \dfrac{2x-1}{3x+2}$ 22. $\lim_{x\to-\infty} \dfrac{4x^2+5}{x^2+3}$

23. $\lim_{x\to\infty} \dfrac{x}{x^2-1}$ 24. $\lim_{x\to-\infty} \dfrac{5x^3+1}{10x^3-3x^2+7}$

25. $\lim_{x\to-\infty} \dfrac{x}{\sqrt{x^2-x}}$ 26. $\lim_{x\to-\infty} \dfrac{x}{\sqrt{x^2+1}}$

27. $\lim_{x\to-\infty} \dfrac{2x+1}{\sqrt{x^2-x}}$ 28. $\lim_{x\to\infty} \dfrac{5x^2+2}{\sqrt{x^2+3}}$

29. $\lim_{x\to\infty} \dfrac{\sqrt{x^2-1}}{2x-1}$ 30. $\lim_{x\to-\infty} \dfrac{\sqrt{x^4-1}}{x^3-1}$

31. $\lim_{x\to\infty} \dfrac{x+1}{(x^2+1)^{1/3}}$ 32. $\lim_{x\to-\infty} \dfrac{2x}{(x^6-1)^{1/3}}$

33. $\lim_{x\to\infty} \dfrac{1}{2x+\sin x}$ 34. $\lim_{x\to\infty} \cos\dfrac{1}{x}$

35. $\lim_{x\to\infty} \dfrac{\sin 2x}{x}$ 36. $\lim_{x\to\infty} \dfrac{x-\cos x}{x}$

37. $\lim_{x\to\infty} (2 - 5e^{-x})$ 38. $\lim_{x\to\infty} \dfrac{8}{4-10^{-x/2}}$

39. $\lim_{x\to\infty} \log_{10}(1 + 10^{-x})$ 40. $\lim_{x\to\infty} \left(\dfrac{5}{2} + \ln\dfrac{x^2+1}{x^2}\right)$

41. $\lim_{t\to\infty} (8t^{-1} - \arctan t)$ 42. $\lim_{u\to\infty} \operatorname{arcsec}(u+1)$

Find the Error In Exercises 43 and 44, describe and correct the error when finding the limit.

43. $\lim_{x\to\infty} \dfrac{5x^3}{6x^3-5} = \lim_{x\to\infty} \dfrac{5}{6-5/x^3} = \dfrac{5}{6-5} = \dfrac{5}{1} = 5$

44. $\lim_{x\to\infty} \dfrac{4x}{\sqrt{x^2+8}} = \lim_{x\to\infty} \dfrac{4x/x}{\sqrt{x^2+8}/\sqrt{x^2}} = \lim_{x\to\infty} \dfrac{4}{\sqrt{1+8/x^2}} = \dfrac{4}{3}$

Horizontal Asymptotes In Exercises 45–48, use a graphing utility to graph the function and identify any horizontal asymptotes.

45. $f(x) = \dfrac{|x|}{x+1}$ 46. $f(x) = \dfrac{|3x+2|}{x-2}$

47. $f(x) = \dfrac{3x}{\sqrt{x^2+2}}$ 48. $f(x) = \dfrac{\sqrt{9x^2-2}}{2x+1}$

Finding a Limit In Exercises 49 and 50, find the limit. (*Hint:* Let $x = 1/t$ and find the limit as $t \to 0^+$.)

49. $\lim_{x\to\infty} x\sin\dfrac{1}{x}$ 50. $\lim_{x\to\infty} x\tan\dfrac{1}{x}$

Finding a Limit In Exercises 51–54, find the limit. (*Hint:* Treat the expression as a fraction whose denominator is 1, and rationalize the numerator.) Use a graphing utility to verify your result.

51. $\lim_{x\to-\infty} \left(x + \sqrt{x^2+3}\right)$ 52. $\lim_{x\to\infty} \left(x - \sqrt{x^2+x}\right)$

53. $\lim_{x\to-\infty} \left(3x + \sqrt{9x^2-x}\right)$ 54. $\lim_{x\to\infty} \left(4x - \sqrt{16x^2-x}\right)$

Numerical, Graphical, and Analytic Analysis In Exercises 55–58, use a graphing utility to complete the table and estimate the limit as x approaches infinity. Then use a graphing utility to graph the function and estimate the limit. Finally, find the limit analytically and compare your results with the estimates.

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

55. $f(x) = x - \sqrt{x(x-1)}$ 56. $f(x) = x^2 - x\sqrt{x(x-1)}$

57. $f(x) = x\sin\dfrac{1}{2x}$ 58. $f(x) = \dfrac{x+1}{x\sqrt{x}}$

WRITING ABOUT CONCEPTS

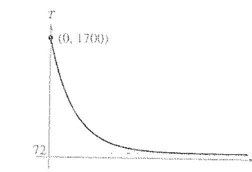
59. **Writing** In your own words, describe what is meant by the statements (a) $\lim_{x\to\infty} f(x) = 4$ and (b) $\lim_{x\to-\infty} f(x) = 2$.

60. **Writing** In your own words, state the guidelines for finding the limit of a rational function. Give examples.

WRITING ABOUT CONCEPTS (continued)

61. **Writing** Consider the function $f(x) = \dfrac{2}{1+e^{1/x}}$.
 (a) Use a graphing utility to graph f.
 (b) Write a short paragraph explaining why the graph has a horizontal asymptote at $y = 1$ and why the function has a nonremovable discontinuity at $x = 0$.

62. **HOW DO YOU SEE IT?** The graph shows the temperature T, in degrees Fahrenheit, of molten glass t seconds after it is removed from a kiln.



 (a) Find $\lim_{t\to 0^+} T$. What does this limit represent?
 (b) Find $\lim_{t\to\infty} T$. What does this limit represent?
 (c) Will the temperature of the glass ever actually reach room temperature? Why?

Comparing Functions In Exercises 63 and 64, (a) use a graphing utility to graph f and g in the same viewing window, (b) verify algebraically that f and g represent the same function, and (c) zoom out sufficiently far so that the graph appears as a line. What equation does this line appear to have? (Note that the points at which the function is not continuous are not readily seen when you zoom out.)

63. $f(x) = \dfrac{x^3-3x^2+2}{x(x-3)}$, $g(x) = x + \dfrac{2}{x(x-3)}$

64. $f(x) = -\dfrac{x^3-2x^2+2}{2x^2}$, $g(x) = -\dfrac{1}{2}x + 1 - \dfrac{1}{x^2}$

65. **Engine Efficiency** The efficiency of an internal combustion engine is

$$\text{Efficiency (\%)} = 100\left[1 - \frac{1}{(v_1/v_2)^c}\right]$$

where v_1/v_2 is the ratio of the uncompressed gas to the compressed gas and c is a positive constant dependent on the engine design. Find the limit of the efficiency as the compression ratio approaches infinity.

66. **Average Cost** A business has a cost of $C = 0.5x + 500$ for producing x units. The average cost per unit is $\bar{C} = C/x$. Find the limit of $\bar{C}$ as x approaches infinity.